OF
AIRCRAFT OF RELAXED STATIC STAB L TY
W TH
PITCH ATTITUDE FEEDBACK

by

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INDIAN INSTITUTE OF TECHNOLOGY, KANPUR

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OF
AIRCRAFT OF RELAXED STATIC STABILITY
WITH
PITCH ATTITUDE FEEDBACK

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CERTIFICATE

This is to certify that this work entitled "A Study of the Longitudinal Dynamical Characteristics of Aircraft of Relaxed Static Stability with Pitch Attitude Feedback" has been carried out by S.GOPALAKRISHNA under my supervision and that this has not been submitted elsewhere for obtaining a degree.

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Symbol	Definition	Usual notation
$\bar{c}$	mean aerodynamic chord	ft
g	acceleration due to gravity	$ft\text{-}sec^{-2}$
h	altitude	ft
$i_B = I_{yy}$	moment of inertia about Y-axis	$slug\text{-}ft^2$
m	mass of the aircraft	slug
M	moment about Y-axis	$slug\text{-}ft^2$
n	load factor	-
Q_1	steady state pitch rate	$rad\text{-}sec^{-1}$
q	perturbed pitch rate	$rad\text{-}sec^{-1}$
S	wing plan area	ft^2
s	root of characteristic equation of longitudinal motion	sec^{-1}
$T_{1/2}$	time to halve the amplitude	sec
T_2	time to double the amplitude	sec
U_1	steady state forward velocity	$ft\text{-}sec^{-1}$
u	perturbed forward velocity	$ft\text{-}sec^{-1}$
W	weight of the aircraft	lbs
X	force along X-axis	slug $ft\text{-}sec^{-2}$
$\bar{X}_{c.g}$	c.g location of the aircraft	ft
Z	force along Z-axis	slug $ft\text{-}sec^{-2}$
C_L	lift coefficient of aircraft	-
C_Z	force coefficient of aircraft along Z-axis	-

Definition

drag coefficient of aircraft

force coefficient of aircraft along

X-axis

thrust coefficient

pitching moment coefficient of air-craft

lift coefficient for zero angle of attac

drag coefficient for zero angle of attack

pitching moment coefficient for zero

angle of attack

aircraft lift curve slope

variation of force coefficient along

Z-axis with angle of attack

variation of drag coefficient with angle
attack

variation of force coefficient with

angle of attack along x-axis

variation of X-thrust coefficient with

angle of attack

variation of pitching moment coefficient

with angle of attack

variation of thrust pitching moment with
of attack

variation of lift coefficient with

elavator angle

variation of drag coefficient with elevat

angle

variation of pitching moment coefficient

with elevator angle

Symbol	Definition
c_{m_θ}	variation of pitching moment coefficient with pitch attitude
c_{L_u}	variation of lift coefficient with non-dimensionalized speed
c_{z_u}	variation of force coefficient along Z-axis with non-dimensionalized speed
c_{D_u}	variation of drag coefficient with non-dimensionalized speed
c_{x_u}	variation of force coefficient along x-axis with non-dimensionalized speed
$c_{T_{x_u}}$	variation of X-thrust coefficient with non-dimensionalized speed
c_{m_u}	variation of pitching moment coefficient with non-dimensionalized speed
$c_{m_{T_u}}$	variation of thrust pitching moment with non-dimensionalized speed
$c_{L_{\dot\alpha}}$	variation of lift coefficient with non-dimensionalized rate of change of angle of attack
$c_{Z_{\dot\alpha}}$	variation of force coefficient along Z-axis with non-dimensionalized rate of change of angle of attack
$c_{m_{\dot\alpha}}$	variation of pitching moment coefficient with non-dimensionalized rate of angle of attack

Symbol	Definition
c_{L_q}	variation of lift coefficient with non-dimensionalized pitch rate
c_{Z_q}	variation of force along X-axis with non-dimensionalized pitch rate
c_{D_q}	variation of drag coefficient with non-dimensionalized pitch rate
c_{m_q}	variation of pitching moment coefficient with non-dimensionalized pitch rate
X_α	dimensional variation of X_s-force with angle of attack (see Table 2.2)
X_{δ_e}	dimensional variation of X_s-force with elevator angle (see Table 2.2)
X_u, X_{T_u}	dimensional variation of X_s-force with speed (see Table 2.2)
Z_α	dimensional variation of Z_s-force with angle of attack (See Table 2.2)
Z_{δ_e}	dimensional variation of Z_s-force with elevator angle (see Table 2.2)
Z_u	dimensional variation of Z_s-force with non-dimensionalized speed (see Table 2.2)
$Z_{\dot{\alpha}}$	dimensional variation of Z_s-force with rate of change of angle of attack (see Table 2.2)
Z_q	dimensional variation of Z_s-force with pitch rate (see Table 2.2)

Symbol	Definition
M_α, M_{T_α}	dimensional variation of pitching moment with angle of attack (see Table 2.2)
$M_{\delta e}$	dimensional variation of pitching moment with elevator angle (see Table 2.
M_θ	dimensional variation of pitching moment with pitch attitude
$M_{\dot{\alpha}}$	dimensional variation of pitching moment with non-dimensionalized rate of change of angle of attack (see Table 2.
M_q	dimensional variation of pitching moment with non-dimensionalized pitch rate (see Table 2.2)
α_1	steady state angle of attack
α	perturbed angle of attack
δ_e	elevator angle
θ_1	steady state pitch attitude angle
θ	perturbed pitch attitude angle
ρ	air density
$\mu = \frac{2m}{\rho S\bar{c}}$	aircraft density factor
ζ_{sp}	short period damping
ζ_p	phugoid or long period damping
$\omega_{n_{sp}}$	undamped natural frequency
ω_{n_p}	undamped plugoid or long period frequency

SYNOPSIS

The thesis entitled, "A Study of the Longitudinal Dynamical Characteristics of Aircraft of Relaxed Static Stability with Pitch Attitude Feedback" is submitted in partial fulfilment of the requirements for degree of M.Tech. by S. GOPALAKRISHNA to the Department of Aeronautical Engineering, Indian Institute of Technology, Kanpur in June 1988.

The longitudinal dynamical characteristics of aircraft of relaxed static stability, equipped with pitch attitude feedback to the elevator are studied in comparison with conventional aircraft. Relaxation of longitudinal static stability of aircraft is known to eliminate the trim drag and thus increase the overall aerodynamic efficiency of the aircraft and thereby reduce the operating costs. However such aircraft require stability augmentation. Pitch attitude feedback is one such means of stability augmentation.

The dynamical characteristics are studied in the cruise, climb and dive configurations and in a symmetrical vertical loop maneuver for an example aircraft by

1) computation of the roots of the characteristic equation.

ii) determination of the response of the aircraft to perturbation in state and control by direct solution of the equations of motion.

iii) a preliminary sensitivity analysis.

The results of the above studies show that pitch attitude feedback to the elevator (or alternatively, effective pitch stiffness) uniformly increases short-period frequency, suppresses long-period oscillations and improves long-period damping. At nominal values of static stability (or equivalently angle of attack stiffness), pitch attitude feedback decreases short-period damping while for a aircraft of relaxed static stability, the short-period damping is increased.

Noting the similarities of the effects of M_α and M_θ and that the sum of the two stiffness is a measure of total effective static stability, the study of the dynamical characteristics is made with several combinations of angle of attack stiffness, and effective pitch stiffness keeping the total effective static stability constant.

A distribution factor K, defined such that

$$K = M_\theta / M_{\alpha_o}$$
$$(1-K) = \frac{M_\alpha}{M_{\alpha_o}}$$

M_{α_o} is the angle of attack stiffness for the conventional aircraft.

The dynamical characteristics with variation of K were studied. The aircraft is said to be superaugmented when K = 1. K = 0 refers to conventional aircraft.

The effect of superaugmentation in cruise, climb and dive and in vertical loop maneuver on short-period frequency is perceptible decrease in frequency and significant decrease in damping. In cruise and climb, the effect of pitch attitude feedback on long-period mode was suppression of oscillations with perceptible deterioration of critical mode (least stable mode). However for a value of distribution factor K between 0 and 1, the long-period oscillation is suppressed and damping significantly improved. This should be the optimal choice of K in the cruise and climb configuration. In the dive configuration pitch attitude feedback attenuates the divergence of the long-period mode and strengthens the convergent root.

To summarise, in cruise, climb and dive configuration, the effect of superaugmentation is favourable on the long-period modes and moderately unfavourable to short-period damping.

In vertical loop maneuver, pitch attitude feedback suppresses long-period oscillation and increases its stability significantly.

Noting that a good dynamical system would be invariant with respect to variation in parameters imposed on it from within or without, a sensitivity study of the dynamical characteristics with respect to aerodynamic derivatives and such other parameters is made.

It may be noted that the roots of the characteristic equation are functions of the coefficients of the characteristic polynomial, which in turn are functions of aerodynamic derivatives and flight condition.

The expressions for non-normalized sensitivity functions of roots of characteristic equation with respect to angle of attack stiffness (static stability), pitch damping and aircraft density factor were obtained.

The sensitivity functions of roots of characteristic equation with respect to angle of attack stability and pitch damping were extracted from the root locus studies described earlier by finite difference method for conventional and superaugmented aircraft. The sensitivity of the roots of the dynamical equations were in general lower for the superaugmented aircraft to the conventional aircraft.

CHAPTER

INTRODUCTION

The present work is an attempt to study the longitudinal dynamical characteristics of superaugmented aircraft (i.e. aircraft with relaxed static stability, augmented with pitch attitude feedback) in comparison with conventional aircraft using root locus and time response methods.

1.1 Historical Background

Automatic control systems were used to compensate the stability of the aircraft as aircraft of early period (1910) were marginally stable. As World War I resulted in aircraft with inherently stable airframes, control system played its part in providing relief to the pilot during long flights. The tremendous increase in aircraft performance was accompanied by continual decrease in airframe inherent stability.

This resulted in aircraft designed from performance point of view only with stability and control provided artificially through automatic control system. This has led to Active Control Technology which is applied for not only stability augmentation and flight control, but explicit towards improvements of the performance of the aircraft. These include drag reduction, gust alleviation, improved handling and riding qualities, flutter suppression, improved

aerodynamic efficiency and load distribution and maneuverabi-
lity.

1.2 Active Control Technology (ACT)

Active control technology involves continuous measurement of several flight variables via sensors installed at selected stations on the aircraft, complex processing of signals obtained from the sensors and feed them back to actuate various aerodynamic surfaces to achieve the required objective.

Before the application of ACT to flight control systems became a reality, availability of highly sensitive yet accurate sensors and transducers, the electronic hardware to process the signals and compact, powerful and fast control surface actuators - all developed to a high degree of reliability was necessary. For, ACT is applied in some very critical regimes of operation of aircraft involving their safety. One of the methods of improving performance of aircraft is trim drag reduction on the tail surface by relaxing the static stability of the airframe, the stability augmented artificially.

1.3 Relaxed Static Stability

At high flight speeds, in aircraft with normal tail plane configuration, the tail is subjected to downward lift in order to produce positive pitching moment normally

required for trim. This is aerodynamically inefficient and results in what is called trim drag. It is possible to have an upward load on the tailplane for trim at low lift coefficients eliminating trim drag and thus improving the aerodynamic efficiency, by relaxation of static stability. This may be done by moving the c.g of the aircraft rearwards, by reducing the tail size, and in several other ways.

The implications in terms of operating costs, of improvements of aerodynamic efficiency of aircraft are quite obvious. The relaxing of static stability provides the added advantages of increased maneuverability.

The price of better performance and maneuverability achieved through relaxed static stability is the poor handling qualities (if not uncontrollability) of the aircraft due to possible short-period divergence. Also tail size reductions may accompany low levels of short-period damping. An aircraft of relaxed static stability needs high gain large bandwidth controllers for stabilization [1]. The short-period divergence and low levels of short-period damping may be improved by augmenting the stability derivatives.

The most obvious stability derivatives to improve static stability and damping are M_α and M_q respectively. Sensing angle of attack and pitch rate and giving a feedbac

to the elevator will augment M_α and M_q.

Table 1.1 indicates some elementary feedback control possibilities including what is termed as superaugmentation to correct the deficiencies of aircraft with relaxed static stability.

1.4 Superaugmented Aircraft

Superaugmented aircraft as defined in [1] are those aircraft which are

i) Statically unstable without augmentation.

ii) Have a degree of pitch attitude stability with respect to inertial space ($M_\theta < 0$) (as opposed to weather cock stability $M_\alpha < 0$) which is provided by the flight control system.

iii) Have pilot command/aircraft pitch response characteristics that are largely independent of aerodynamic derivatives.

Superaugmented aircraft are an important sub-class of actively controlled, highly augmented aircraft. These aircraft perhaps bear such a name because the control system not only augments the stability but also brings out a change in effective vehicle dynamics that differ in kind from those associated with conventional aircraft [1].

The advantage of superaugmentation is that it is possible to optimize the aircraft configuration without any need for compromise with stability and control requirements other than the provision of adequate control power [1]. The disadvantages of superaugmentation are

i) when control surfaces are momentarily saturated, the aircraft will be unstable and will tend to diverge until control can be restored.

ii) complexity of flight control system

iii) cost.

Furthermore, the flying qualities near the limits of controller effectiveness differ markedly from the conventional aircraft [2]. The deteoriation in flying qualitie due to time lag although these can be improved by use of properl designed pre-filters [2],[3],[5] and [6].

The reasons for using pitch attitude feedback system instead of many other feedback systems(Table 1.1) is the ease of implementation. To redress static instability, feedback of angle of attack would be an effecti method. However, measuring angle of attack and providing redundancy in the measurement by the sensor leads to implementation difficulties [3]. Measuring pitch rate is simpler and by providing an integrator in the sens

change in pitch attitude is directly measured. The origin of this flight control came in a way to discourage the use of air-data, encourage the use of inertial sensors for the sole purpose of redundancy and ruggedness [2] . These deficiencies are redressed by augmenting stability through $\int$ qdt-δ feedback control system.

1.5 Preview of the Work

In this work a superaugmented aircraft is referred to as an aircraft with fully relaxed static stability ($M_\alpha = 0$) and having pitch attitude feedback to elevator through constant gain.

Of the three primary functions of superaugmented aircraft namely -

i) stability augmentation,
ii) improved flying qualities and
iii) insensitivity of dynamical characteristics to variation in flight conditions and aerodynamic derivatives.

In this work the first function is addressed to and an exploratory investigation into the last is made. The performance of superaugmented aircraft in several longitudinal symmetrical flight configurations is evaluated.

Chapter 2 describes the scope and method of study dynamical characteristics of longitudinal motion of an

aircraft with variation of aerodynamic derivatives. The particulars of the representative aircraft on which the above study is made are also described. An analytical study of sensitivity of roots of the characteristic equation of longitudinal motion, to variation in aerodynamic derivatives is made later in the chapter. The computational procedures used for the above studies are described at the end of the chapter.

Chapter 3 discusses the results obtained from the computation of roots to variations in some aerodynamic derivatives, control parameter and of time response to angle of attack perturbation and to step elevator input. The results of sensitivity of the roots to variation in the aerodynamic derivatives about their nominal value is also discussed later in the chapter.

Chapter 4 lists the conclusions, and suggestions for future work.

CHAPTER 2

THEORY AND COMPUTATIONS

2.1 Overview

In this chapter, the method of studying the variatio of the dynamical characteristics of the longitudinal motion of an aircraft with respect to variations of some aerodynami derivatives and of the feedback control parameter is outline The particulars of a representative aircraft on which the above studies are made are given briefly. An analytical study of the sensitivity of the roots of the characteristic equation of longitudinal motion with variation of some aerodynamic derivatives and flight condition has been made.

2.2 Equations of Motion

2.2.1 Rectilinear Flight

The longitudinal perturbation equations of motion in steady rectilinear flight of a conventional aircraft under the usual assumptions of small perturbations and linearity (of perturbed force and moment coefficients as a functions of perturbed variables of motion) are well known and may be found in standard textbooks on Dynamics of Flight [4],[8].

These equations with reference to stability axes are given below in dimensional form:

$$\dot{u} = X_u u + X_{T_u} u + X_\alpha \alpha - g \cos\theta_1 \theta + X_{\delta_e} \delta_e$$

$$U_1(\dot{\alpha} - q) = Z_u u + Z_\alpha \alpha + Z_{\dot{\alpha}} \dot{\alpha} + Z_q q - g \sin\theta_1 \theta + Z_{\delta_e} \delta_e$$

$$\dot{q} = M_u u + M_{T_u} u + M_\alpha + M_{T_\alpha} + M_{\dot{\alpha}} \dot{\alpha} + M_q q + M_{\delta_e} \delta_e$$

$$\dot{\theta} = q \qquad (2.1)$$

where the dimensional derivatives are defined as indicated below:

$$X_u = \frac{1}{m} \frac{\partial x}{\partial u}; \quad X_\alpha = \frac{1}{m} \frac{\partial x}{\partial \alpha};$$

$$Z_u = \frac{1}{m} \frac{\partial z}{\partial u}; \quad Z_\alpha = \frac{1}{m} \frac{\partial z}{\partial \alpha}; \quad Z_{\dot{\alpha}} = \frac{1}{m} \frac{\partial z}{\partial \dot{\alpha}} \text{ etc}$$

and

$$M_u = \frac{1}{I_{yy}} \frac{\partial M}{\partial u}; \quad M_\alpha = \frac{1}{I_{yy}} \frac{\partial M}{\partial \alpha}; \quad M_{\delta_e} = \frac{1}{I_{yy}} \frac{\partial M}{\partial \delta_e} \text{ etc.}$$

$$M_{T_u} = \frac{1}{I_{yy}} \frac{\partial M_T}{\partial u} \qquad M_{T_\alpha} = \frac{1}{I_{yy}} \frac{\partial M_T}{\partial \alpha}$$

where U_1 and θ_1 are respectively the airspeed and pitch angle in the reference flight condition; $u, \alpha,$ θ, q and δ_e are the perturbation velocity, angle of attack, pitch attitude, pitch rate and elevator angle respectively.

The expressions for the above dimensional derivative in terms of nondimensional aerodynamic derivatives and flight conditions are given in Table 2.1.

However, for aircraft provided with pitch attitude feedback to the elevator, the equations have to be suitably modified. It must be noted that the proposed feedback to the elevator is not the absolute value of pitch attitude, but the perturbation value with reference to the instantaneous pitch attitude in the reference trajectory of the aircraft in the flight configuration considered. We may observe that the introduction of pitch attitude feedback to the elevator with gain K is equivalent to the introduction of

i) an effective pitch attitude stiffness M_θ such that $M_\theta = M_{\delta_e} K$ where M_{δ_e} is the elevator power.

ii) an effective normal force derivative Z_θ such that $Z_\theta = Z_{\delta_e} K$ where Z_{δ_e} is the normal force derivati with respect to elevator deflection.

iii) an effective axial force derivative X_θ such that $X_\theta = X_{\delta_e} K$, where X_{δ_e} is the axial force derivative with respect to elevator.

Therefore, the equation of motion of conventional aircraft with pitch attitude feedback would take the form,

$$\dot{u} = X_u u + X_{T_u} u + X_\alpha \alpha - g \cos\theta_1 \theta + X_\theta \theta + X_{\delta_e} \delta_e$$

$$U_1 (\dot{\alpha} - q) = Z_u u + Z_\alpha \alpha + Z_{\dot{\alpha}} \dot{\alpha} + Z_q q - g \sin\theta_1 \theta + Z_\theta \theta + Z_{\delta_e} \delta_e$$

$$\dot{q} = M_u u + M_{T_u} u + M_\alpha \alpha + M_{T_\alpha} \alpha + M_{\dot{\alpha}} \dot{\alpha} + M_q q + M_\theta \theta + M_{\delta_e} \delta_e$$

$$\dot{\theta} = q . \qquad (2.2)$$

However, since the effect of the elevator on the pitching of an aircraft is significantly more predominant in comparison with that on normal forces and axial forces, the effective force derivatives Z_θ and X_θ are ignored. Now it remains to add the term $M_\theta \theta$ to the pitching moment equation of motion of conventional aircraft.

With the above approximation, the equation of motion of conventional aircraft with pitch attitude feedback reduces to the following:

$$\dot{u} = X_u u + X_{T_u} u + X_\alpha \alpha + X_{\delta_e} \delta_e$$

$$U_1 (\dot{\alpha} - q) = Z_u u + Z_\alpha \alpha + Z_{\dot{\alpha}} \dot{\alpha} + Z_q q - g \sin\theta_1 \theta + Z_{\delta_e} \delta_e$$

$$\dot{q} = M_u u + M_{T_u} u + M_\alpha \alpha + M_{T_\alpha} \alpha + M_{\dot{\alpha}} \dot{\alpha} + M_q q + M_\theta \theta + M \delta_e \delta_e$$

$$\dot{\theta} = q \qquad (2.3)$$

2.2.2 Vert al Loo Mane er

The perturbation equations of motion of the aircraft in Maneuvers are not so well known as those in rectilinear flight. We consider for our study of longitudinal dynamics, a steady symmetrical vertical loop maneuver at constant load factor and constant speed.

The equations in the above vertical loop maneuver for a conventional aircraft are, noting that in the reference trajectory $\dot{U}_1 \equiv \dot{W}_1 \equiv W_1 \equiv 0$

$$X + X_T - g \, \mathrm{Sin}\theta_1 = m(\dot{U}_1 + Q_1 W_1) = 0$$

$$Z + g \, \mathrm{Cos}\,\theta_1 = m(\dot{W}_1 - Q_1 U_1) = m(Q_1 U_1)$$

$$M + M_T = I_{yy} \dot{Q}_1 \qquad (2.4)$$

The pitch rate Q_1 of an aircraft is related to the speed U_1, the normal load factor n and the pitch attitude θ_1 by the following relation:

$$Q_1 = g(n - \mathrm{Cos}\theta_1)/U_1 \qquad (2.5)$$

By inspection of equation (2.4), perturbation equations may be written as

$$X_u u + X_{T_u} u + X_\alpha \alpha + X_{\delta_e} \delta_e - g \, Cos\theta_1 \theta = \dot{u} + Q_1 W_1 + qW_1$$
$$= \dot{u} + Q_1 U_1 \alpha$$

$$Z_u u + Z_\alpha \alpha + Z_{\dot{\alpha}} \dot{\alpha} + Z_q q - g \, Sin\theta_1 \theta + Z_{\delta_e} \delta_e = \dot{w} - Q_1 u - qU_1$$

$$M_u u + M_{T_u} u + M_\alpha \alpha + M_{T_\alpha} \alpha + M_{\dot{\alpha}} \dot{\alpha} + M_q q + M_{\delta_e} \delta_e = \dot{q} \qquad (2.6)$$

The additional term Q_1U_1 and Q_1u appearing in the pertur-tion equation in a vertical loop Maneuver, compared to the corres equations in rectilinear motion may be treated as equivalent to modifying the term Z_α and Z_u of the X and Z equations in the rectilinear motion by an amount of Q_1U_1 and Q_1 respectively.

2.3 Methods of Study

The study of longitudinal dynamical characteristics of the conventional aircraft and aircraft provided with pitch attitude feedback including the superaugmented aircraft in various flight configuration are proposed to be studied by the following two methods:

i) By computation of roots of characteristic equation.

ii) By computation of time response of variables of motion such as velocity, angle of attack, pitch rate and load factor to an initial perturbations in any one of the above variables.

It is further proposed to extend the above studies to varying values of stability derivatives such as M_α, M_q and the parameters of feedback control such as M_θ primarily to determine the effect of above parameters of the aircraft on its dynamical characteristics.

The study was further extended to the various flight configuration of aircraft such as cruise, steep climb, dive and vertical loop maneuver.

With the help of above studies, a direct computation of the sensitivity of dynamical characteristics of the aircraf to variations in several parameters was made with a view to find out to what extent the dynamics of superaugmented aircraf are insensitive to variations of the aerodynamic derivatives {9].

If an attempt to explore the possibilities of achievin the much desired insensitivity of aircraft dynamics with variation in principal aerodynamic derivatives and in flight conditions through appropriate feedback control, an analytical study of the above problem has been made.

The above studies were made using an example aircraft. The aircraft chosen for this purpose was a conventional Buisness jet transport aircraft for which a case for superaugementation exists from considerations of improving the economy of operations via reduction of trim drag. The details of the aircraft such as the values of the stability

derivatives etc. required for the proposed studies were taken from [4] . These are shown in Table 2.2 under cruise configuration.

While extending our proposed studies to steep climb and dive and symmetrical vertical loop maneuver, some of the stability derivatives which have values different from those given for cruise condition had to be estimated by a method which involves certain approximations and these values are shown in Table 2.2 under the corresponding configuration.

For instances in addition to cruise configuration at a velocity 675 ft/sec, and lift coefficient of 0.410, a steady climb configuration at a lift coefficient which is the same as in cruise was chosen. The climbing flight therefore takes place at lower velocity than the cruise and at lower Mach no. The above choice of flight configuration involving preservation of the lift coefficient facilitated easy and accurate extraction of dimensional derivatives from those available for the cruise configuration excepting for the compressibility effect which was neglected.

For a climb angle of 60° for which the computations were made, the Mach no falls from 0.7 in cruise to 0.5 in climb.

These studies were extended to steady dive at the same lift coefficient and speed as in climb.

A steady vertical loop maneuver at constant velocity and constant load factors of 4 was chosen for the study of the dynamics. Here the speed was chosen to be the same as in cruise, while the life coefficient increased to 4 times the cruise value. In vertical loop maneuver, the values of some aerodynamic derivatives differ from those in cruise. These values were estimated for the example aircraft from a parabolic approximation made to the drag polar given in graphical form in Ref. [4].

It must be noted that the vertical loop maneuver may be a hypothetical one for the example aircraft, which is a business transport aircraft. However such aircraft are generally stressed to a normal load factor of 3.5g.

The dynamics of the aircraft are studied in a vertical loop maneuver at two cardinal points of a circular loop viz. at the bottom and the top.

In the perturbation equations of motion for the vertical loop maneuver (equation 2.6), the values of Θ_1 will be 0° and 180° at bottom and top respectively and the values of Q_1 will be $\frac{3g}{U_1}$ and $\frac{5g}{U_1}$ by virtue of equation (2.5).

The variation of the dynamical characteristics of the aircraft with respect to some important parameters including aerodynamic derivatives, feedback gain, and flight conditions, is studied over a range of values of the parameters about the nominal values.

For instance, the nominal value of angle of attack stiffness M_α for the example aircraft is -7.5. The above studies are made over a range of M_α varying from -15 to +15. The nominal value of pitch damping M_q is -0.94, the range was chosen from -2.0 to 0.0.

Normalised to their respective nominal values the range of the aerodynamic derivatives, considered will be +2 to -2 for M_α ; -2.0 (approximately) to 0.0 for M_q.

For the pitch attitude feedback gain expressed in terms of effective pitch stiffness M_θ, the range of normalized values is from -2 to +2. Preliminary studies have sho as one may anticipate, the pitch stiffness M_θ augments the angle of attack stiffness M_α so far as the short-period dynamical behaviour is concerned.

It is proposed to keep the sum of M_α and M_θ constan at the nominal value of M_α of the aircraft and vary its distribution factor between M_α and M_θ. A distribution fact K is defined thus

$$K = \frac{M_\theta}{M_{\alpha_o}}$$

it follows that $1-k = \dfrac{M_\alpha}{M_{\alpha_o}}$

where M_{α_o} is the nominal value of M_α for the aircraft. For the example aircraft, $M_{\alpha_o} = -7.5$. Thus for K=0, $M_\alpha = M_{\alpha_o}$ and $M_\theta = 0$ which describes the conventional

aircraft without feedback. For K=1, $M_\alpha = 0$ and $M_\theta = M_{\alpha_0}$. This represents an aircraft of relaxed static stability with pitch attitude feedback fully compensating for the static instability. We designate such an aircraft as the superaugmented aircraft. For all other values of K, $M_\alpha = (1-K) M_{\alpha_0}$ and $M_\theta = K M_{\alpha_0}$.

As stated in Chapter 1, the dynamical characteristics of the aircraft are studied by

i) computation of the roots of characteristic equation

ii) obtaining the response of the aircraft to small perturbations in angle of attack and to a step input of elevator.

It is well known that the characteristic equation of conventional aircraft in longitudinal motion may be written in the form

$$f(s) = As^4 + Bs^3 + Cs^2 + Ds + E = 0 \qquad (2.7)$$

A,B,C,D and E may be expressed in terms of aerodynami derivatives, flight conditions in general. The expressions for these coefficients may be found in standard text-books on flight dynamics [4],[8]. Even if a pitch attitude feedback is incorporated in the aircraft, one may see that the characteristic polynomial f(s) remains a quartic. The expressions for the coefficients of the characteristic polyno

for an aircraft with pitch attitude feedback are given in Appendix A.

Primarily with a view to study the sensitivity of the longitudinal dynamics of the aircraft with respect to variation in derivatives M_α, M_θ and M_q, a root locus study of the characteristic equation was made for variation of the above parameters over the range described earlier. This study was done only for the cruise configuration of the airplane the results of which are shown from Figure 3.1 to Figure 3.5.

The root locii were drawn for variation of distributio factor K over a range -1 to +1.

The sensitivity of the dynamical characteristics of the aircraft (as reflected in roots of the characteristic equation), with respect to the derivatives M_α, M_q and M_θ are determined from the root locus using finite difference method. This was done for the conventional aircraft and superaugmented aircraft in cruise configuration [9]. The results of the above study is tabulated in Table 3.2.

2.4 Analytical Study of Sensitivity

An analytical study of sensitivity of roots of the characteristic equation with respect to aerodynamic derivatives and flight conditions is attempted with a view to determine the conditions on the superaugmented control of

the air craft required for achieving low values of sensitivity of the roots.

The sensitivity of the root s_j with respect to the aerodynamic derivative P_k is defined here in non-normalized form as follows:

$$S_j = \frac{\partial s_j}{\partial P_k}$$

$$\frac{\partial s_j}{\partial P_k} = \sum_{i=0}^{N} \frac{\partial s_j}{\partial C_i} \cdot \frac{\partial C_i}{\partial P_k} = \sum_{i=0}^{N} \frac{\partial s_j}{\partial f} \cdot \frac{\partial f}{\partial C_i} \cdot \frac{\partial C_i}{\partial P_k}$$

writing $\frac{\partial s_j}{\partial f} = \frac{1}{\partial f / \partial s_j}$

we have

$$\frac{\partial s_j}{\partial P_k} = \frac{1}{\partial f / \partial s_j} \cdot \sum_{i=0}^{N} \frac{\partial f}{\partial C_i} \cdot \frac{\partial C_i}{\partial P_k}$$

From equation (2.7), we have

$$f = \sum_{0}^{4} C_i \, s^4 = As^4 + Bs^3 + Cs^2 + Ds + E$$

$$\frac{\partial s_j}{\partial P_k} = \left. \frac{s^4 \cdot \frac{\partial A}{\partial P_k} + s^3 \cdot \frac{\partial B}{\partial P_k} + s^2 \cdot \frac{\partial C}{\partial P_k} + s \cdot \frac{\partial D}{\partial P_k} + \frac{\partial E}{\partial P_k}}{4As^3 + 3Bs^2 + 2Cs + D} \right|_{s=s_j} \quad (2.8)$$

a. The Sensitivity of Root with Variation in M_α

The sensitivity of roots of characteristic equation of longitudinal motion of conventional aircraft with pitch attitude feedback to variation in M_α is given by

$$\frac{\partial s_j}{\partial M_\alpha} = \left. \frac{s^2(U_1 + Z_q) + s(X_u + X_{T_u})(U_1+Z_q) + (gZ_u - (X_u+X_{T_u}))}{4As^3 + 3Bs^2 + 2Cs + D} \right|_{s=s_j} \tag{2.9}$$

as $\frac{\partial A}{\partial M_\alpha} = 0$, $\frac{\partial B}{\partial M_\alpha} = 0$

b. The Sensitivity of Roots with Variation in M_q

The sensitivity of roots of characteristic equation of longitudinal motion of conventional aircraft with pitch attitude feedback with variation in M_q is given by

$$\frac{\partial s_j}{\partial M_q} = \frac{-s^3(U_1 - Z_{\dot\alpha}) + s^2((X_u+X_{T_u})(U_1 - Z_{\dot\alpha}) + Z_\alpha) + s(X_\alpha Z_u) - Z_q(X_u+X_{T_u})}{4As^3 + 3Bs^2 + 2Cs + D} \tag{2.10}$$

c. The Sensitivity of Roots with Variation in Aircraft Density Factor μ

The sensitivity of roots of characteristic equation of longitudinal motion of conventional aircraft with pitch

$$\frac{\partial s_j}{\partial \mu} = \frac{\begin{aligned} & s^4(8\mu i_B - 2C_{z_\alpha} i_B) + s^3(-2i_B(C_{z_\alpha} + C_{x_u}) - 2(C_{z_q} C_{m_{\dot\alpha}} \\ & -C_{m_q} C_{z_{\dot\alpha}}) - 8\mu(C_{m_{\dot\alpha}} + C_{m_q})) + s^2(2(C_{z_\alpha} C_{m_q} \\ & -C_{m_\alpha} C_{z_q} + C_{x_u} C_{m_q} + C_{x_u} C_{m_{\dot\alpha}}) - 8\mu C_{m_\theta} + \\ & 2 C_{z_{\dot\alpha}} C_{m_\theta} - 8\mu C_{m_\alpha}) + s(2(C_{x_u} C_{m_\alpha} - C_{x_\alpha} C_{m_u} + C_{L_o} C_{m} \\ & -2C_{z_\alpha} C_{m_\theta} + 2 C_{x_u} C_{m_\theta}) \end{aligned}}{4\bar{A}s^3 + 3\bar{B}s^2 + 2\bar{C}s + D}$$

where the expressions for coefficients $\bar{A}$, $\bar{B}$, $\bar{C}$, $\bar{D}$ are given in Appendix A in terms of non-dimensional derivatives.

In order that an aircraft have satisfactory dynamical characteristics over the range of its operation, it is not only necessary that the roots of the characteristic equation should lie within prescribed values but they also should be insensitive with respect to each of the flight conditions. These conditions may in principle be satisfied if there are enough control parameters that will be adjusted so as to meet this conditions as best as possible. The control parameters in an aircraft provided with feedback control system can be the gains of the various feedback loops incorporated in the control system.

The dynamical characteristics of the conventional aircraft (K=0) and of the superaugmented aircraft (K=1) are studied by obtaining the response of the aircraft to perturbations in angle of attack in five configurations namely cruise, steep climb, dive and a vertical loop maneuver (top and bottom). The response to elevator input was obtained for the cruise configuration.

2.5 Computational Procedure

The roots of the characteristic equation of longitudinal motion were computed using NAG standard Libr sub-routine CO2AEF which is based on Grant-Hitchin method The computation of the response of the aircraft required solution of the ordinary differential equation of motion using the NAG sub-routine DO2BBF which is based on Runge-Kutta Merson Method. Appendix B gives the listing of computer programs used. The samples of responses obtaine by this method were tested against those obtained by the transfer function decomposition.

CHAPTER 3

RESULTS AND DISCUSSIONS

In this chapter, the results of the computations, presented in figures and tables are discussed. The dynamical characteristics of the aircraft are extracted from time response plots and from root locii. The effect of superaugmentation on the dynamical characteristics is discussed. A comparative study of the results obtained for the five configurations is made. A summary of the results is presented in Table 3.1.

The root locus with variation of M_α is shown in Fig. 3.1 for the conventional aircraft in cruise configurati the angle of attack stability M_α is relaxed from -7.5 to 0.0, the short-period mode degenerates from oscillatory convergence to subsidence. The long-period mode deteriorate from near oscillatory neutral stability to oscillatory divergence. Interestingly, when M_α is made positive, one long-period mode becomes stable and the other degenerates to short-period divergence. Also, one short-period mode degenerates to long-period divergence.

Figure 3.2 shows the root locus with variation of M_α for the pitch stabilized aircraft (M_θ = -7.5). For the

nominal value of M_α (-7.5), the short-period mode has higher frequency than the normal aircraft, while damping is reduced. The increase in the short-period frequency may be anticipated as the pitch stiffness M_θ may be considered as augmenting the angle of attack stability M_α. The long-period mode which is oscillatory for the conventional aircraft becomes subsident due to the pitch stiffness. As the angle of attack stability is relaxed, the short-period mode is still oscillatory with reduced frequency and damping. The long-period mode is subsident but with reduced stability.

As the angle of attack stability is made negative (M_α = +15), the short-period mode becomes oscillatory diverge with reduced frequency, while the modes corresponding to long-period oscillation for the conventional aircraft degenera to one heavy subsidence and one light divergence. Increasing angle of attack stability (M_α = -15) increases the short-period frequency and damping while both long-period modes are subsident.

From a comparison of Fig. 3.1 and Fig. 3.2, it is evident that the effect of introducing pitch stiffness on the dynamics of the airplane is as though the angle of attack stiffness is augmented.

The root locus to variation in M_q for conventional aircraft is shown in Fig. 3.3. The short-period damping deteriorates as M_q varies from nominal value of -0.941 to 0.0 whereas the frequency nearly remains invariant. The long-period mode is nearly insensitive to variation in M_q.

Figure 3.4 shows the root locus to variation in M_q for superaugmented aircraft. The short-period mode is the same as conventional aircraft and long-period mode is insensitive to variation in M_q as in conventional aircraft.

Figure 3.5 shows the root locus with variation in M_θ for aircraft with relaxed static stability ($M_\alpha = 0.0$) As the pitch stiffness is reduced from -15.0 to -7.5, the short-period damping which is moderate remains unaffected while the frequency decreases considerably. As pitch stiffness is further reduced to zero ($M_\theta = 0$), short-perioc frequency and damping are reduced and a short-period mode degenerates to long-period mode of slight divergent oscill tions. If the pitch stiffness is made negative ($M_\theta = +15.($ the mode tends to light subsidence and strong divergence.

The long-period mode for $M_\theta = -15.0$ consists of moderate subsidence and a very light subsidence. For the superaugmented aircraft configuration, we have moderately damped short-period and heavily damped long-period subsidence.

As the pitch stiffness is decreased, and further made negative ($M_\theta = +15$), both the above modes becomes increasingly subsident. It is interesting to note that while the angle of attack stability is fully relaxed ($M_\alpha = 0.0$) and pitch stiffness is made negative ($M_\theta = +15.0$) the modes which were originally of long-period becomes increasingly stable, although one of two short-period mode disintegrates into strong divergence.

The root locus for variation of distribution factor is shown in Fig. 3.6 for the example aircraft in cruise configuration. It may be observed that for the conventional aircraft, (K=0), the short-period mode is moderately damped while the long-period mode is nearly neutrally stable. Superaugmentation (K=1) results in reduced damping of the short-period mode, leaving the frequency unaffected, while the oscillatory long-period mode degenerates to a mode of moderate convergence and slight divergence. On the other hand when K = -1, the short-period damping increases while the long-period mode degenerates to two divergent modes.

We may conclude from the above that the short-period frequency is nearly invariant with K over the range (-1 to +1) considered where in the sum of M_α and M_θ is conserved. The loss of M_α is nearly fully compensated for by M_θ of equal magnitude as far short-period frequency is concerned.

In so far as the short-period damping concerned, the loss of damping due to relaxed angle of attack stability as the complex roots degenerate to real roots is not fully compensated for by the increase in damping by the introduction of pitch stiffness. This is evident from Fig. 3.1, Fig. 3.5 and Fig. 3.6. It may be noted from the root locus that the best damping in long-period mode is achieved at a value of K intermediate between 0 and 1, where the long-period roots are equal to each other and convergent. The time to halve the amplitude $T_{1/2} = 7.7$ sec.

Figure 3.9 shows the time response of conventional aircraft to unit angle of attack disturbance in cruise configuration. It may be clearly seen that the short-period is fully damped with in 5 seconds and the long-period is nearly undamped. It may also be seen that there is a long-period component of the angle of attack. Thus response curve is in conformity with the results obtained from root locii plot of Fig. 3.1 and Fig. 3.6.

Figure 3.10 shows the time response of superaugmented aircraft to unit angle of attack disturbance in cruise configuration. Short-period mode is clearly discernable in pitch rate response and is damped with in two cycles and 5 seconds. We observe that the perturbation velocity deviate from equilibrium value for large times unlike the other variables. This may be due to the presence of small positive real root as observed in Fig. 3.6 and Fig. 3.2 for the

superaugmented aircraft and the possible significant component of velocity corresponding to that root.

Figure 3.13 and Figure 3.14 show the response of conventional and superaugmented aircraft for unit step elevator input in cruise configuration. This response essentially has the same dynamical characteristics as response to angle attack disturbance except that the short-period mode is clearly discernible for superaugmented aircraft for unit step elevator input.

Figure 3.7 shows root locus with variation of K in climb at 60°. For conventional aircraft, K = 0, the short-period mode is oscillatory convergent while the long-period mode is oscillatory divergent. For the superaugmented aircraft K = 1, the short-period mode is slightly less stable than K=0, the long-period mode is purely divergent, the divergence being greater for conventional aircraft. Comparing climb configuration with cruise configuration, we observe that there is consistent deterioration of stability of aircraft, conventional and superaugmented, in both long-period and short-period mode. While the stability of the short-period modes is decreased (by about 25%) in comparison with cruise in both cases, the long-period modes which were neutrally stable in cruise become unstable in climb. It may be noted from the root locus that similar to case of cruise configuration the best value of K lies between 0 and 1 where the divergent long-period mode becomes non-oscillatory and

neutrally stable (i.e. both roots are nearly at the origin).

The response to unit angle of attack disturbance in climb at 60° are shown in Fig. 3.13 and Fig. 3.14. The long-period response is oscillatory divergent for conventional aircraft and pure divergence by superaugmented aircraft. The long-period roots of the characteristic equation computed show consistent results with the response plot. One may conclude that superaugmention does not improve the stability of aircraft.

Figure 3.8 shows the root locus with variation of K in a dive at 60°. The phugoid oscillation is eliminated even for conventional aircraft but had deteriorated to divergence due to one root lying on the positive side of complex plane and other root contributing for neutral stability. The superaugmented aircraft in dive has nearly the same long-period mode as in climb, one root contributing for subsidence and the other to divergence. The short-period roots are nearly the same for both the aircraft as in climb and further more they are less sensitive to variations in K compared to those in cruise. It appears from the above results that the dynamics of superaugmented aircraft are almost the same whether in climb or dive (keeping C_L constan where as for the conventional aircraft it is evidently not so.

The response to unit angle of attack disturbance are shown in Fig. 3.15 and Fig. 3.16 for conventional and superaugmented aircraft respectively. As in climb, the response is divergent for both the aircraft and thus one may conclude that in dive superaugmentation does not augment the stability. Short-period mode is more visible in conventional aircraft. The results of roots of characteristic equation computed in dive are consistent with response plot.

The time responses of conventional and superaugmented aircraft at the bottom of the vertical loop maneuver is shown in Fig. 3.17 and Fig. 3.18. The response is moderately convergent for conventional aircraft. The response for the superaugmented aircraft appears to be subsident. One may conclude that stability increases with superaugmentation. The roots of the characteristic equation for the correspondin cases has been computed. The time responses obtained are in consistency with the values of roots.

Compared to stability in cruise the stability at the bottom of the vertical loop appears to be greater. This may be anticipated as one may know that the long-period damping is proportional to the ratio of $\frac{C_D}{C_L}$ which is more in loop maneuver, the lift being 4 times that in cruise condition when C_D/C_L is generally minimum.

Figure 3.19 and Fig. 3.20 show the response of the conventional and superaugmented aircraft at the top most point in vertical maneuver. Superaugmentation in this case stabilizes the unstable response. The roots of the characteristic equation for the corresponding cases has been computed. The time responses are in consistence with the value of the roots comparing the result obtained for the response of the conventional aircra in bottom of the loop (Fig. 3.17) to the top of the loop. Fr Figure 3.19 we may observe that although the aircraft opera at identical lift coefficient and velocity, responses are qualitatively different. This should be attributed to the reversal of the gravity force as the aircraft moves from bottom to top of the loop. Superaugmented aircraft shows a similar behaviour. Convergence appears to be same for both bottom and top of the loop. This is confirmed from the values of roots obtained for these cases which were nearly equal.

The results of the above studies are summarised in Table 3.1 which gives the time to halve or double the amplitude computed from the response for large values of time for the several cases and also those computed from root locii for the corresponding cases.

The sensitivity of the roots of the characteristic equation of longitudinal dynamics to variation in some

aerodynamic derivatives such as M_α, M_θ and M_q are computed about their nominal values within a small range from the root locus study by finite difference method. The results are summarised in Table 3.2.

It may be observed from Table 3.2 that the short-period damping is considerably less sensitive to variation in M_α for superaugmented aircraft compared to conventional aircraft whereas there is only slight reduction in the sensitivity of short-period frequency. The phugoid damping is almost insensitive to variation in M_α for the conventiona. aircraft whereas the phugoid frequency is marginally sensitive. For superaugmented aircraft, as long-period root degenerates to real root, the sensitivity of two roots to variation in M_α is marginal.

It may be observed from Table 3.2 that short-period damping is slightly more sensitive to variations in M_q for superaugmented aircraft compared to conventional aircraft, but short-period frequency is relatively insensitive to variation in M_q for superaugmented aircraft. The long-perio damping and frequency are insensitive to variations in M_q for conventional aircraft. The long-period roots of superaugmented aircraft are still more insensitive to variat in M_q.

From Table 3.2, we find that short-period damping is quite insensitive to variations in M_θ whereas short-period frequency is comparatively sensitive about nominal value of $M_\theta = -7.5$. The long-period roots are almost insensitive to variations in M_θ .

In conclusion, one may observe that there is in general improvement of the insensitivity of roots of the longitudinal dynamics with respect to M_α and M_q for the superaugmented aircraft in comparison with the conventional aircraft about their nominal values.

CONCLUSIONS AND SUGGESTIONS FOR FURTHER WORK

In the context of the longitudinal dynamical characteristics of an aircraft, pitch attitude feedback (to the elevator) appears to have the following features relative to angle of attack feedback.

(i) substantially greater effectiveness in suppression of oscillations of the long-period mode,

(ii) slightly better damping of the long-period mode in general,

(iii) almost equal effect on short-period frequency,

(iv) moderately lower sensitivity in general,

(v) substantially greater ease of implementation and ruggedness,

(vi) moderately lower short-period damping.

The clear advantages of the features at (i) and (v) outweigh the disadvantage at (vi) which may be easily remedied by pitch rate feedback to the elevator.

The above features commend pitch attitude feedback as a primary means of stability augmentation of aircraft wi relaxed longitudinal static stability. This may be suppor by angle of attack, pitch rate and velocity feedback where necessary.

Further work in this area may be done as listed below:

i) The studies of the dynamical characteristics of a pitch attitude augmented aircraft may be extended to more symmetric and asymmetric maneuvers.

ii) It would be possible to achieve a set of desired dynamical characteristics and at the same time reduce the sensitivity to variation of flight condition and aerodynamic derivative if one had more control parameters to vary; these could be parameters of filter network or the gains of other types of feedback such as angle of attack, speed, pitch rate, normal acceleration etc. An investigation on the above probl may be done.

iii) Response of pitch attitude feedback system to vertica gusts may be studied by transfer function method in frequency domain and statistical modelling of atmospheric gusts.

iv) The effect of time delay which is inherent in complex digital control systems may be accounted and studied.

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Table 1.1

Some Elementary Feedback Control Possibilities to Correct the Stability Deficiencies of Aircraft with Relaxed Static Stability (Ref. 1)

General effect	Primary effective stability derivatives augmented or created	Feedback control possibilities
Improve short-period damping	$M_{\dot{\theta}}$	Pitch attitude rate $\dot{\theta} - \delta$
	M_q	Pitching velocity $q - \delta$
	$M_{\dot{\alpha}}$	Angle of attack rate $\dot{\alpha} - \delta$
Increase static stability	M_{θ}	Pitch attitude stability $\theta - \delta$
	$M_{\int q}$ (same as M_{θ} when $\phi=0$)	Integral of pitching velocity $\int qdt - \delta$
	$U_o \quad M_{\int az}$	Integral of normal acceleration $\int az\, dt - \delta$
	M_{α}	Angle of attack $\alpha - \delta$
	M_u	Speed $u - \delta$

Table 2.1

Expressions for Longitudinal Dimensional Stability Derivatives

$$X_u = \frac{-\bar{q}_1 s(C_{D_u} + 2C_{D_1})}{m\, U_1}\ \text{sec}^{-1} \qquad M_{T_u} = \frac{\bar{q}_1\ \bar{c}(C_{m_{T_u}} + 2C_{m_{T_1}})}{I_{yy}\ U_1}\ \text{ft}^{-1}\ \text{sec}^{-1}$$

$$X_{T_u} = \frac{\bar{q}_1 s(C_{T_{X_u}} + 2C_{T_{X_1}})}{m\ U_1} \qquad M_\alpha = \frac{\bar{q}_1\ \bar{c}\ C_{m_\alpha}}{I_{yy}}\ \text{sec}^{-1}$$

$$X = \frac{-\bar{q}_1 s(C_{D_\alpha} - C_{L_1})}{m}\ \text{ft sec}^{-2} \qquad M_{T_\alpha} = \frac{\bar{q}_1\ s\ \bar{c}\ C_{m_{T\alpha}}}{I_{yy}}\ \text{sec}^{-2}$$

$$X_{\delta_e} = \frac{-\bar{q}_1 s\ C_{D_{\delta_e}}}{m}\ \text{ft sec}^{-2} \qquad M_{\dot{\alpha}} = \frac{\bar{q}_1 s\ \bar{c}^2\ C_{m_{\dot{\alpha}}}}{2I_{yy}\ U_1}\ \text{sec}^{-1}$$

$$Z_u = \frac{-\bar{q}_1 s(C_{L_u} + 2C_{L_1})}{m\ U_1}\ \text{sec}^{-1} \qquad M_q = \frac{\bar{q}_1\ s\ \bar{c}^2\ C_{m_q}}{2I_{yy}\ U_1}\ \text{sec}^{-1}$$

$$Z_\alpha = \frac{-\bar{q}_1 s\ (C_{L_\alpha} + C_{D_1})}{m}\ \text{ft sec}^{-2} \qquad M_{\delta_e} = \frac{\bar{q}_1 s\ \bar{c}\ C_{m_{\delta_e}}}{I_{yy}}\ \text{sec}^{-2}$$

$$Z_{\dot{\alpha}} = \frac{-\bar{q}_1 s\ C_{L_{\dot{\alpha}}}\ \bar{c}}{2m\ U_1}\ \text{ft sec}^{-1}$$

Continued.....

Table 2.1 (Continued):

$$Z_q = \frac{-\bar{q}_1 \; s \; C_{L_q} \; \bar{c}}{2m \; U_1} \quad \text{ft sec}^{-1}$$

$$Z_{\delta_e} = \frac{-\bar{q}_1 \; s \; C_{L_{\delta_e}} \; \bar{c}}{2m \; U_1} \quad \text{ft sec}^{-2}$$

$$M_u = \frac{\bar{q}_1 \; s \; \bar{c} \; (C_{m_u} + 2C_{m_1})}{I_{yy} \; U_1} \quad \text{ft}^{-1} \; \text{sec}^{-1}$$

$W = 13{,}000$ lbs, $h = 40{,}000$ ft, $= 0.000588$ slug ft^{-3}, $I_{yy} = 18{,}800$ slug ft^2
$M = 0.7$, $\bar{c} = 7.04$ ft, $\bar{X}_{c.g} = 0.315$, $S = 232$ ft, $\theta_1 = 0$ (stability axes)
$\alpha_1 = 2.7$ deg

Non-dimensional derivative	Cruise 675 ft-sec^{-1}	Dimensional derivative	Cruise $U_1 = 675$ ft-sec^{-1}	Climb/Dive 60^o $c_{L_1} = 0.410$	Vertical Loop Maneuver $U_1 = 675$ ft-sec^{-1}
c_{D_u}	0	X_u (sec^{-1})	-0.0075	-0.0053	-0.1057
c_{D_1}	+0.033				
$c_{T_{X_u}}$	0	X_{T_u} (sec^{-1})	+0.0075	+0.05988	+0.0075
$c_{T_{X_1}}$	+0.033				
c_{D_α}	+0.300	X_α (ft-sec^{-2})	+8.46	+4.23	-126.1587
c_{L_1}	0.410				
$c_{D_{\delta_e}}$	0	X_{δ_e} (ft-sec^{-2})	0	0	0
c_{L_u}	+0.400	Z_u (sec^{-1})	-0.139	-0.09828	-0.419
c_{L_α}	+5.84	Z_α (ft-sec^{-2})	-451.7	-285.89	-484.94
$c_{L_{\dot\alpha}}$	+2.20	$Z_{\dot\alpha}$ (ft-sec^{-1})	- 0.882	- 0.6236	- 0.882
c_{L_q}	+4.70	Z_q (ft-sec^{-1})	- 1.885	- 1.333	- 1.885

4

Continued....

$c_{L_{\delta_e}}$	+0.556	Z_{δ_e} (ft-sec^{-2})	-42.776	-21.39	-42.776
c_{m_u}	+0.05	M_u (ft^{-1}sec^{-1})	+ 0.0011	+ 0.00078	+ 0.0011
c_{m_1}	+0.007				
$c_{m_{T_u}}$	-0.0034	M_{T_u} (ft^{-1}sec^{-1})	- 0.0003	- 0.0002	- 0.0003
$c_{m_{T_1}}$	-0.007				
c_{m_α}	-0.64	M_α (sec^{-2})	- 7.448	- 3.724	- 7.448
c_{m_T}	0	M_{T_α} (sec^{-2})	0	0	0
$c_{m_{\dot\alpha}}$	-6.7	$M_{\dot\alpha}$ (sec^{-1})	- 0.407	- 0.288	- 0.407
c_{m_q}	-15.50	M_q (sec ^{1})	- 0.941	- 0.665	- 0.941
$c_{m_{\delta_e}}$	- 1.52	M_{δ_e} (sec^{-2})	-17.689	- 8.845	-17.689

($T_{1/2}$ - Time to halve the amplitude)

T_2 - Time to double the amplitude)

Flight configuration		Time constants by root locus (By time response)			
		Conventional Aircraft		Superaugmented aircraft	
		$T_{1/2}$(sec)	T_2(sec)	$T_{1/2}$(sec)	T_2(sec)
Criuse		1593.44 (1608.2)	-	-	380.25 (371.26)
Climb At 60°		-	13.078 (13.33)	-	12.16 (12.8)
Dive At 60°		-	6.93 (5.76)	-	13.07 (14.14)
Vertical Loop Maneuver	Top	-	10.77 (10.66)	6.41 (6.41)	-
	Bottom	14.35 (14.44)	-	5.649 (5.6)	-

Table 3.2

Summary of the Results of the Sensitivity Studies

Aero dynamical derivatives	Normal configuration $M_\alpha = -7.5,\ M_\theta = 0.0$	Super augmented configuration $M_\alpha = 0.0,\ M_\theta = -7.5$
M_α	$\frac{\partial \zeta_{sp}}{\partial M_\alpha} = 0.024$	$\frac{\partial \zeta_{sp}}{\partial M_\alpha} = +0.0035$
	$\frac{\partial \omega_{nsp}}{\partial M_\alpha} = -0.181$	$\frac{\partial \omega_{nsp}}{\partial M_\alpha} = -0.13$
	$\frac{\partial \zeta_{p}}{\partial M_\alpha} = 0.0002$	$\frac{\partial s_3}{\partial M_\alpha} = 0.0258$
	$\frac{\partial \omega_{np}}{\partial M_\alpha} = 0.0125$	$\frac{\partial s_4}{\partial M_\alpha} = 0.001603$
M_q	$\frac{\partial \zeta_{sp}}{\partial M_q} = -0.1625$	$\frac{\partial \zeta_{sp}}{\partial M_q} = -0.19$
	$\frac{\partial \omega_{ns.p}}{\partial M_q} = -0.12$	$\frac{\partial \omega_{ns.p}}{\partial M_q} = -0.022$
	$\frac{\partial \zeta_{p}}{\partial M_q} = 0.00305$	$\frac{\partial s_3}{\partial M_q} = -0.000002$
	$\frac{\partial \omega_{np}}{\partial M_q} = 0.000425$	$\frac{\partial s_4}{\partial M_q} = -0.0000003$

Continued....

Table 3.2 (Continued):

		$\frac{\partial \zeta_{s.p}}{\partial M_q} = 0.018$
		$\frac{\partial \omega_{ns.p}}{\partial M_\theta} = -0.1945$
M_θ	-	$\frac{\partial s_3}{\partial M_\theta} = 0.0045$
		$\frac{\partial s_4}{\partial M_\theta} = 0.00096$

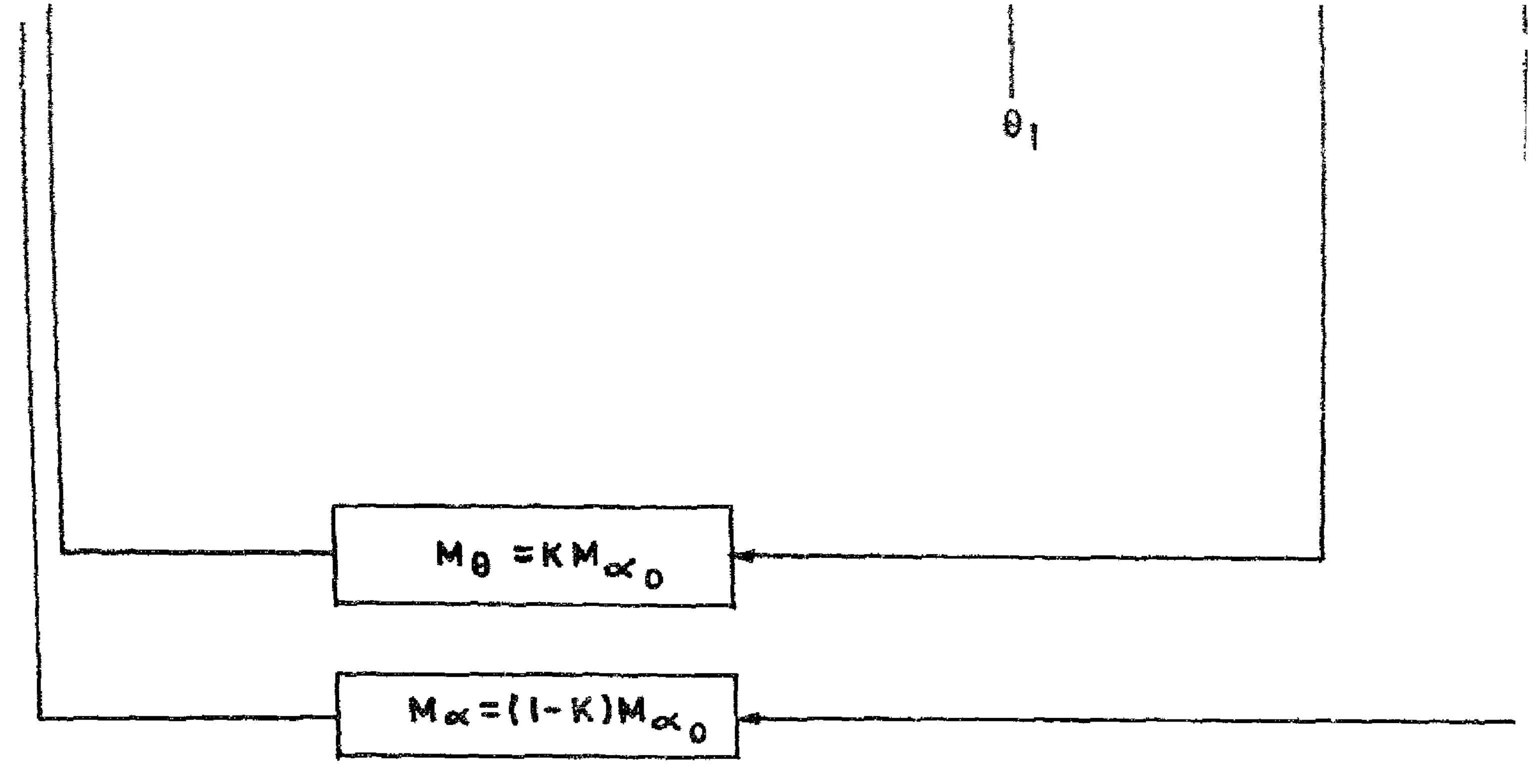


FIG. 3.0 STABILITY AUGMENTATION BY ANGLE OF ATTACK AND PITCH ATTITUDE FEEDBACK

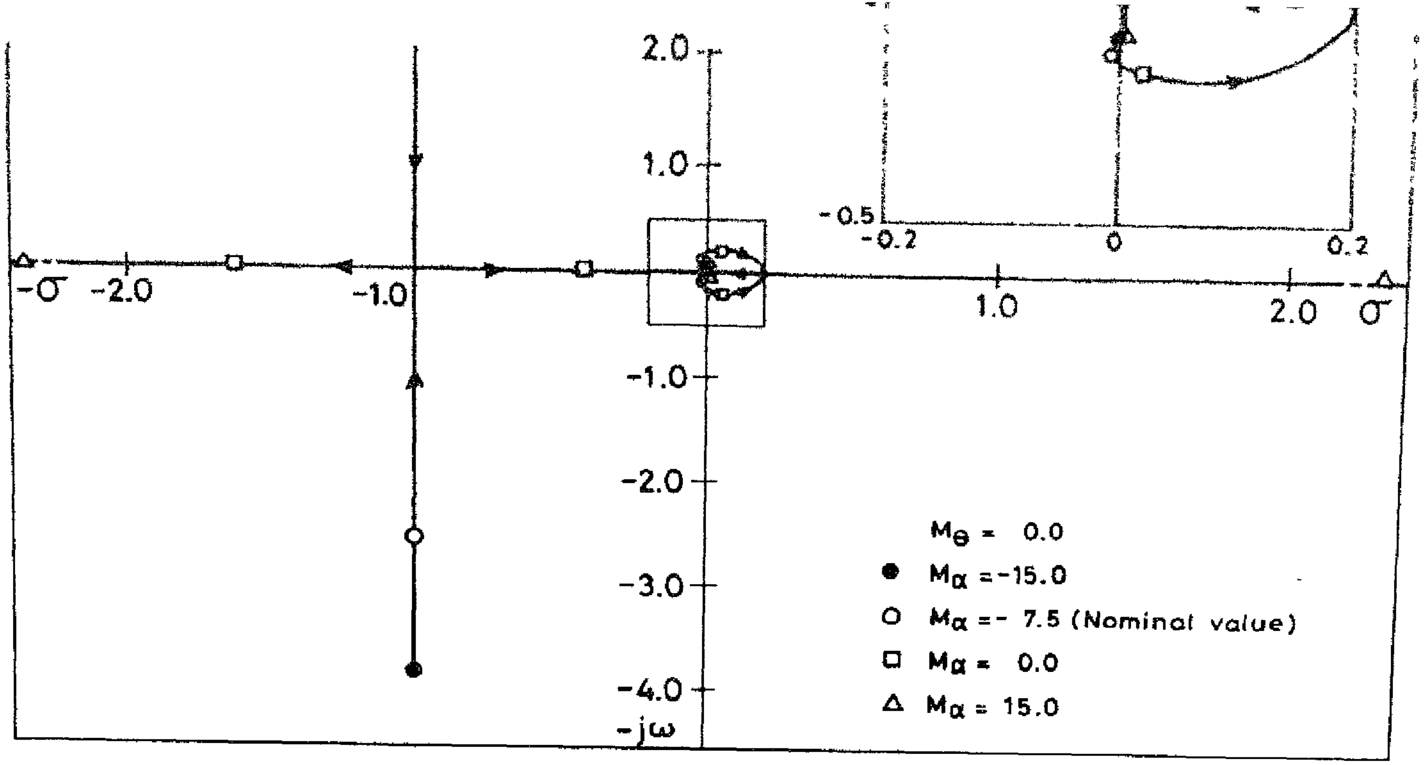


Fig 3.1 Root locus - variation with M_α for conventional aircraft.

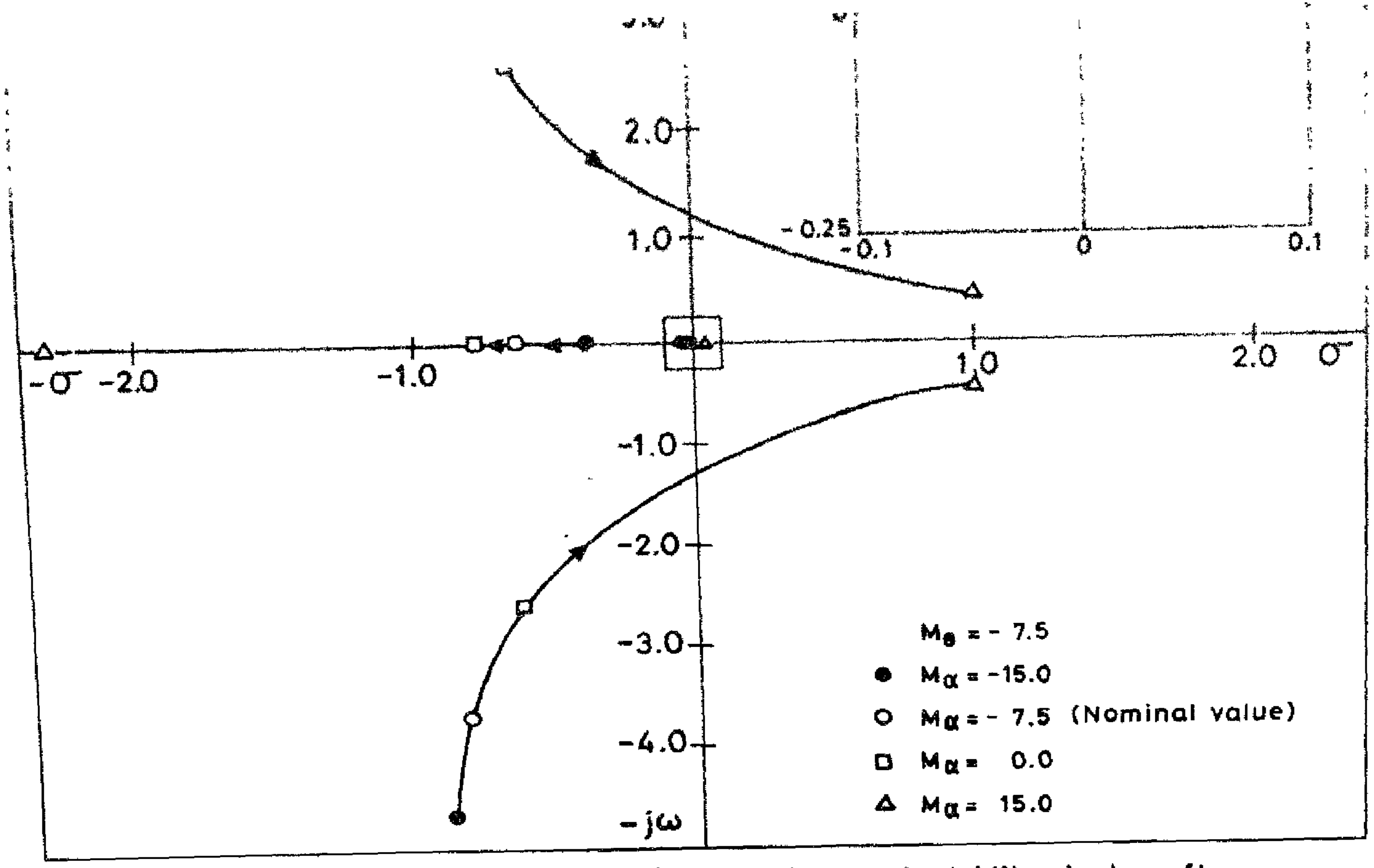


Fig. 3.2 Root locus-variation with M_α for pitch stabilized aircraft

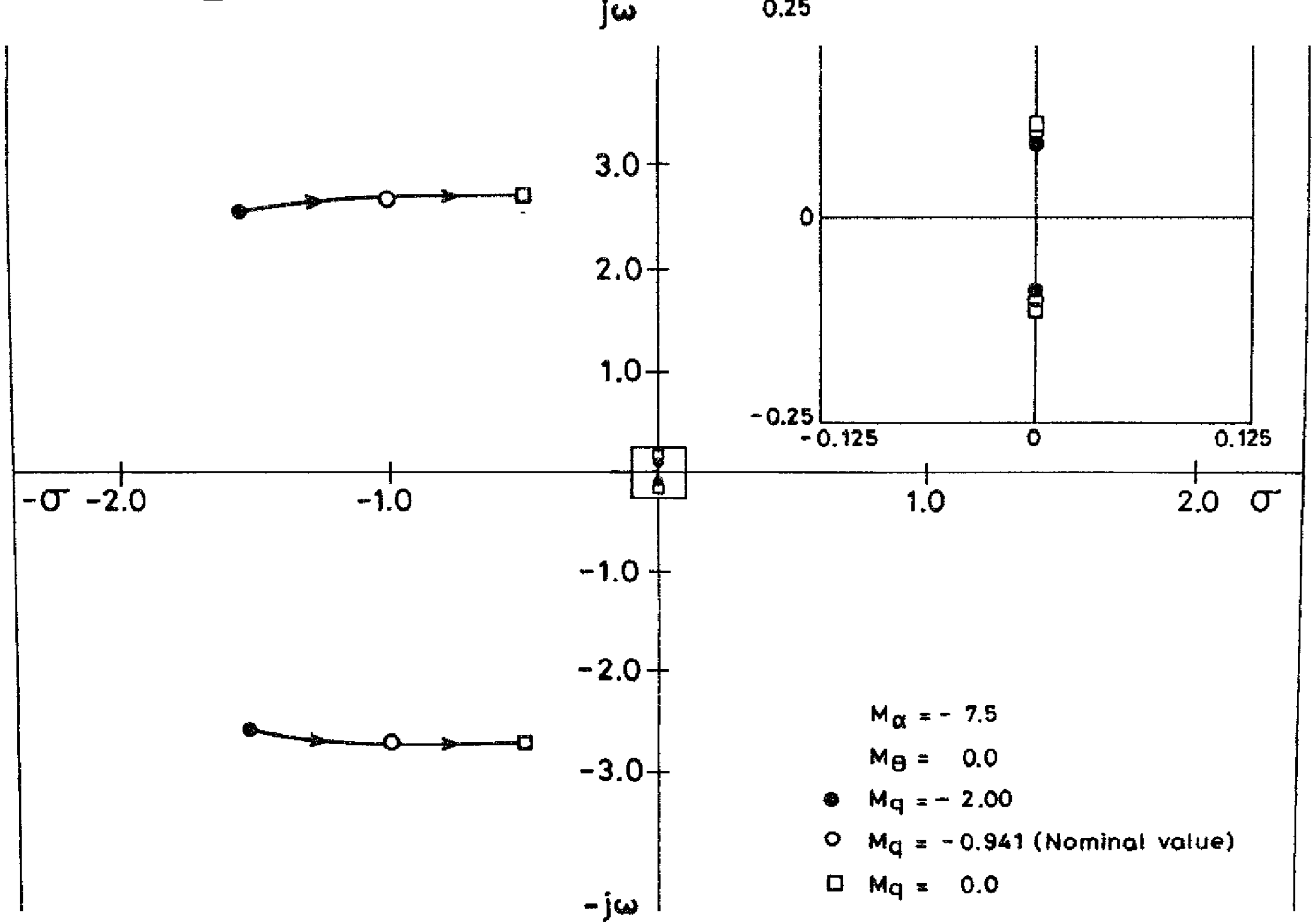


Fig 2.3 Root locus variation with M_q for conventional aircraft

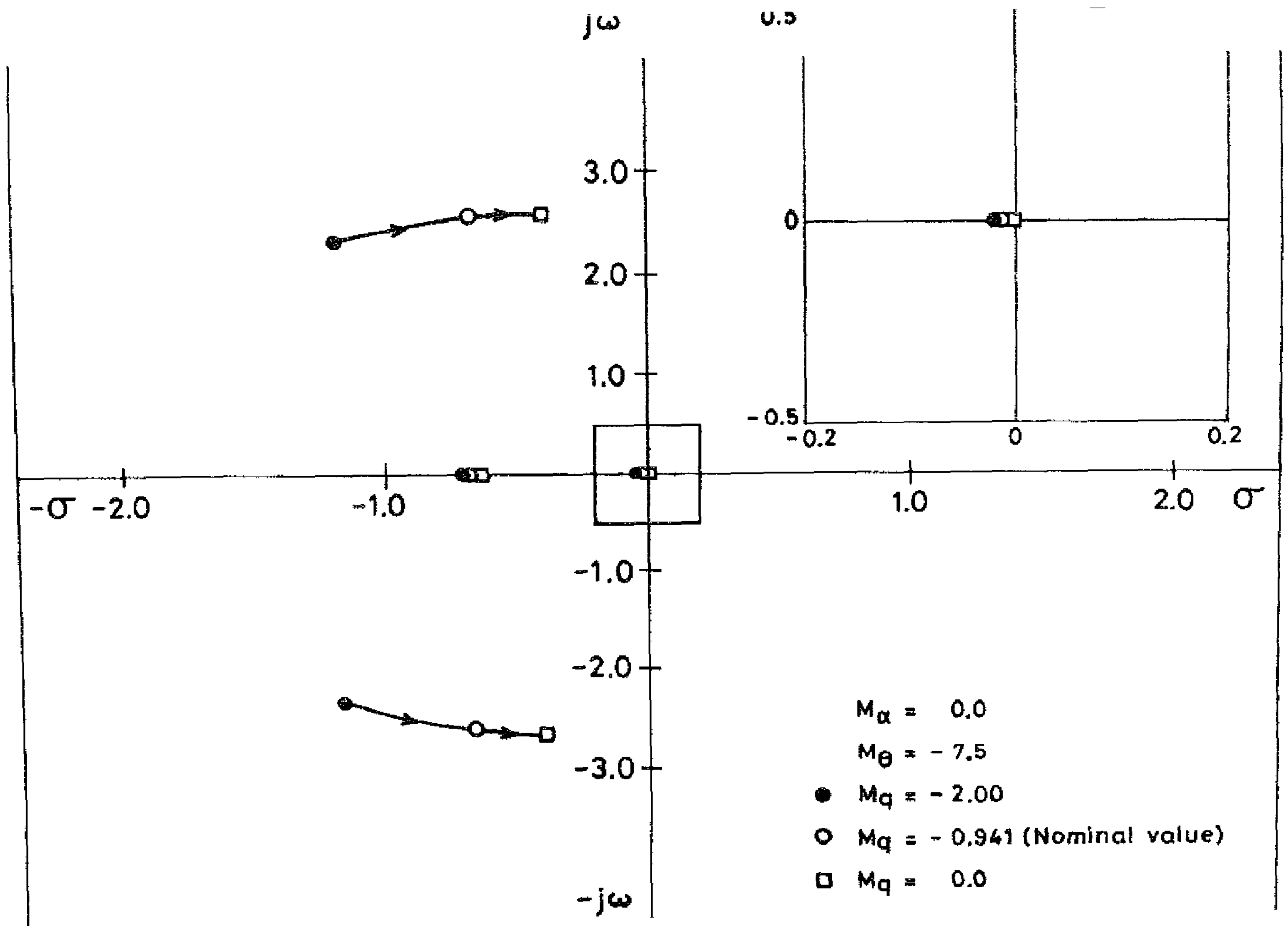


Fig.3.4 Root locus - variation with M_q for super augmented aircraft

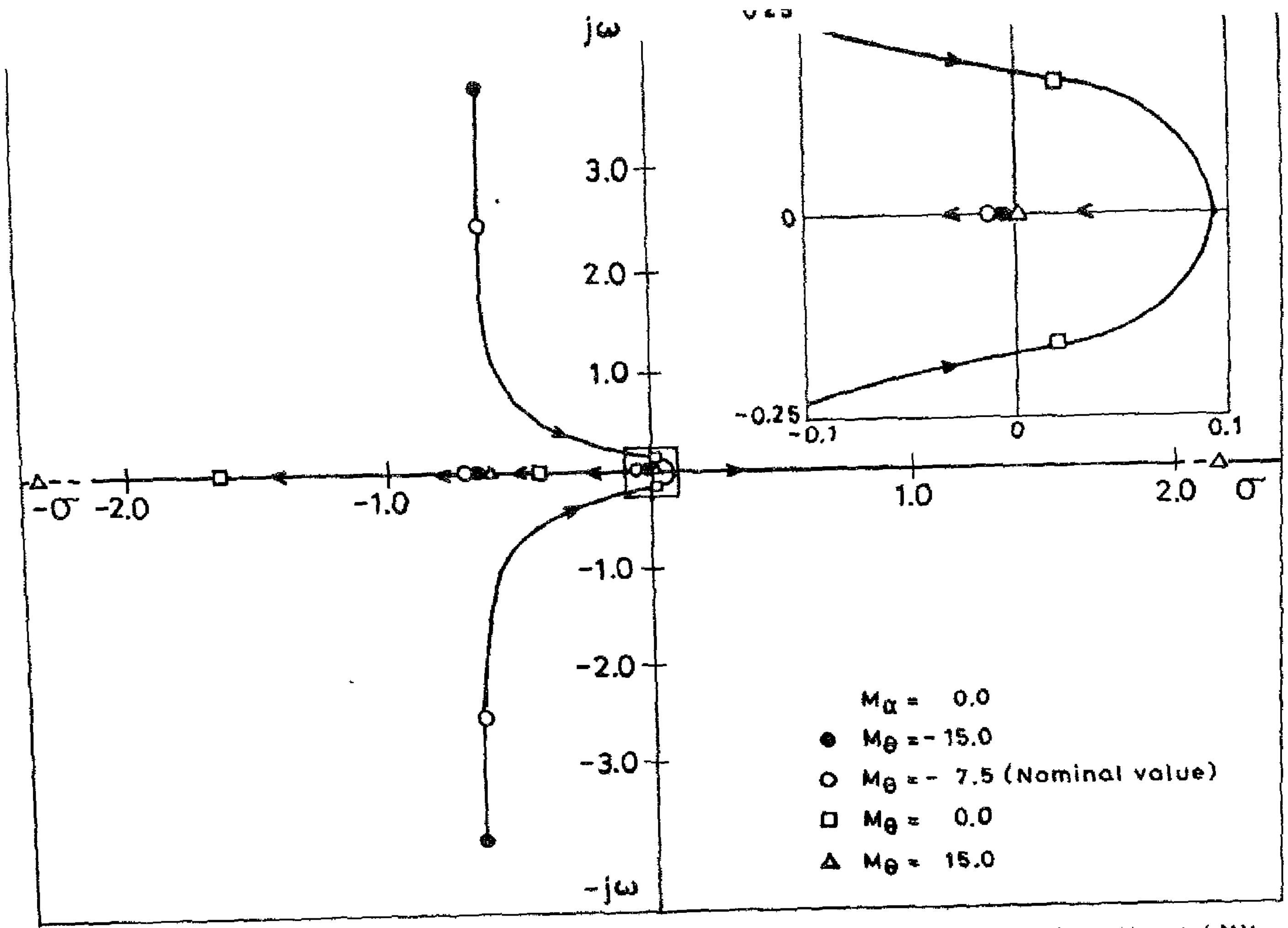


Fig.3.5 Root locus - variation with M_θ for aircraft of relaxed static stability

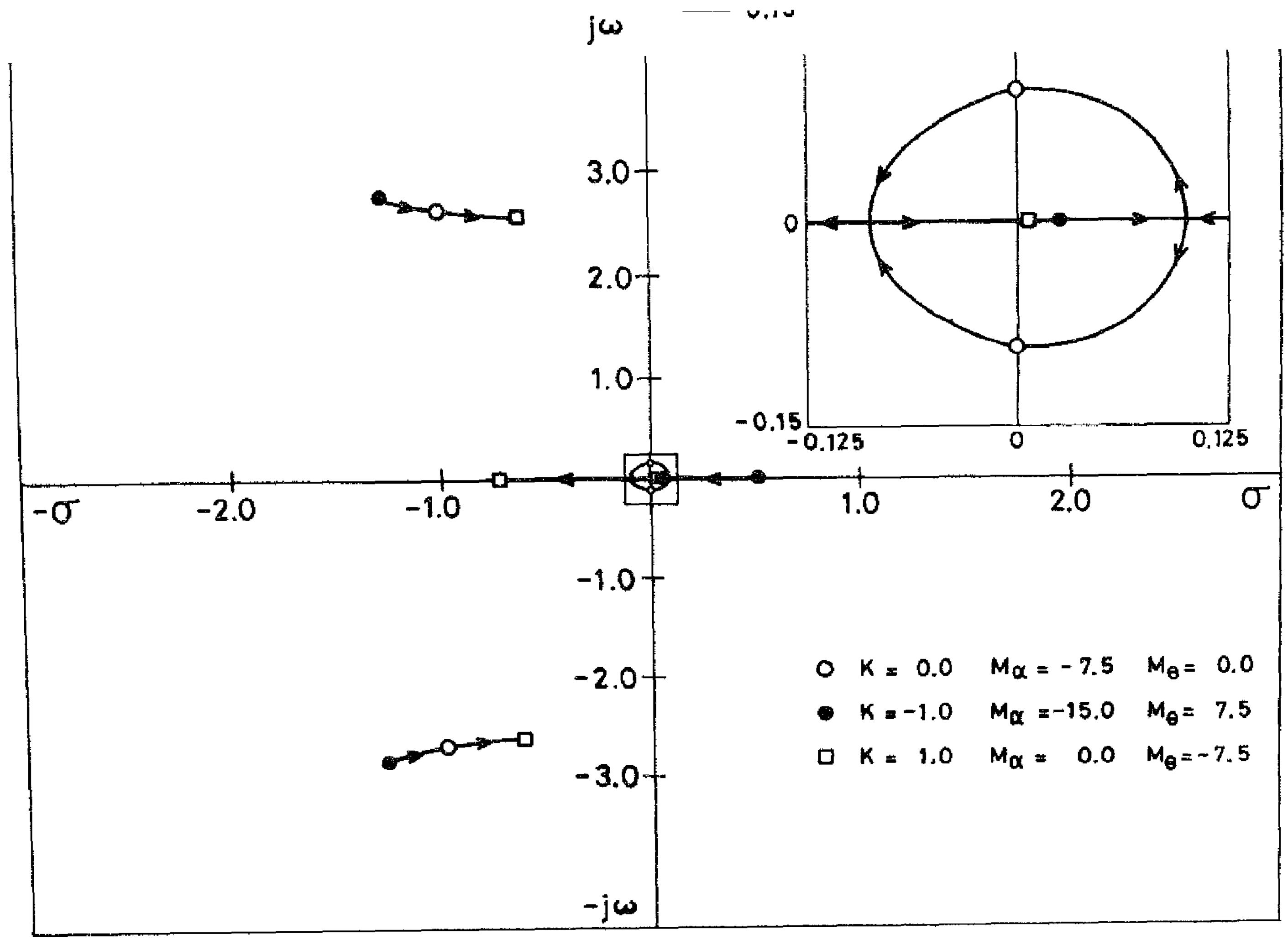


Fig.3.6 Root locus -variation with K in cruise.

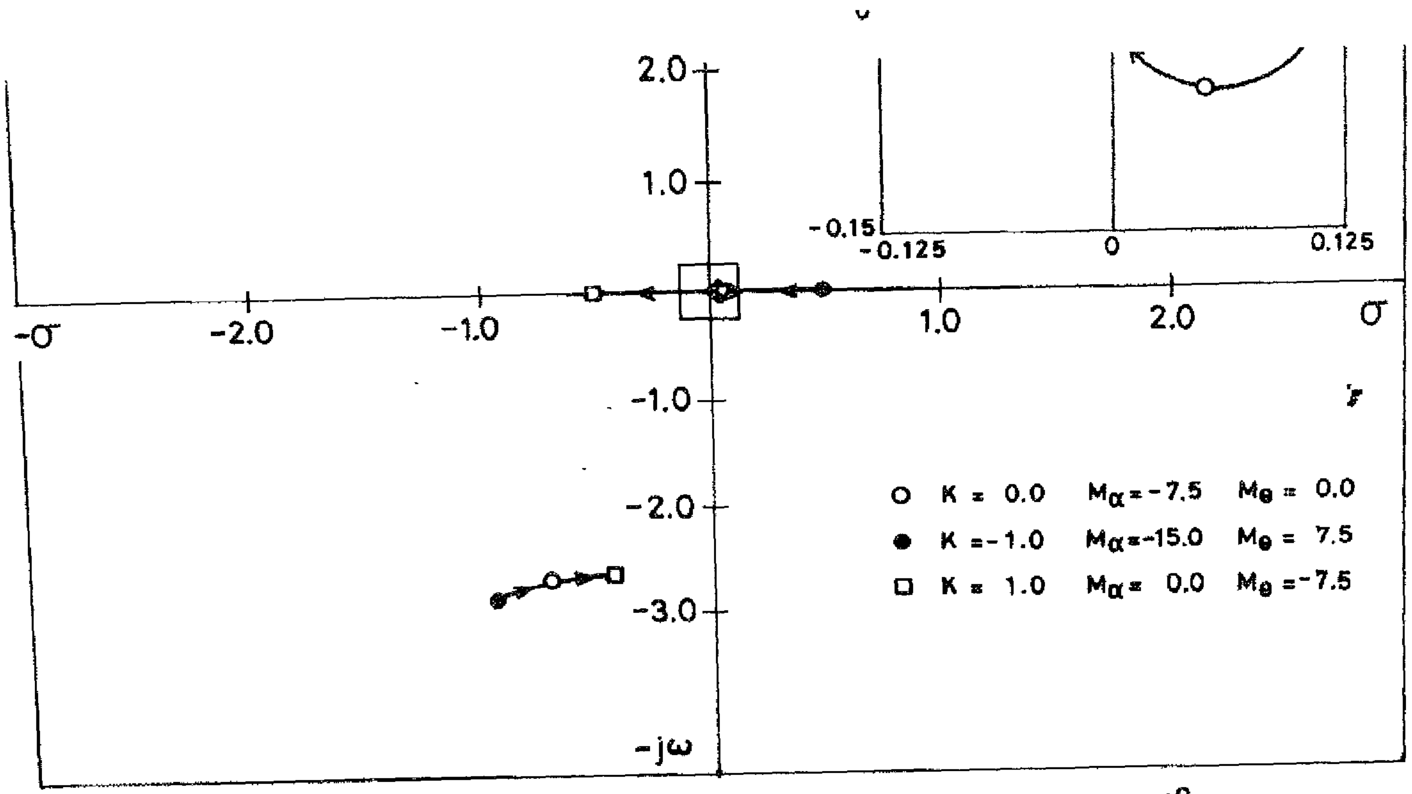


Fig.3.7 Root locus - variation with K in climb at 60°.

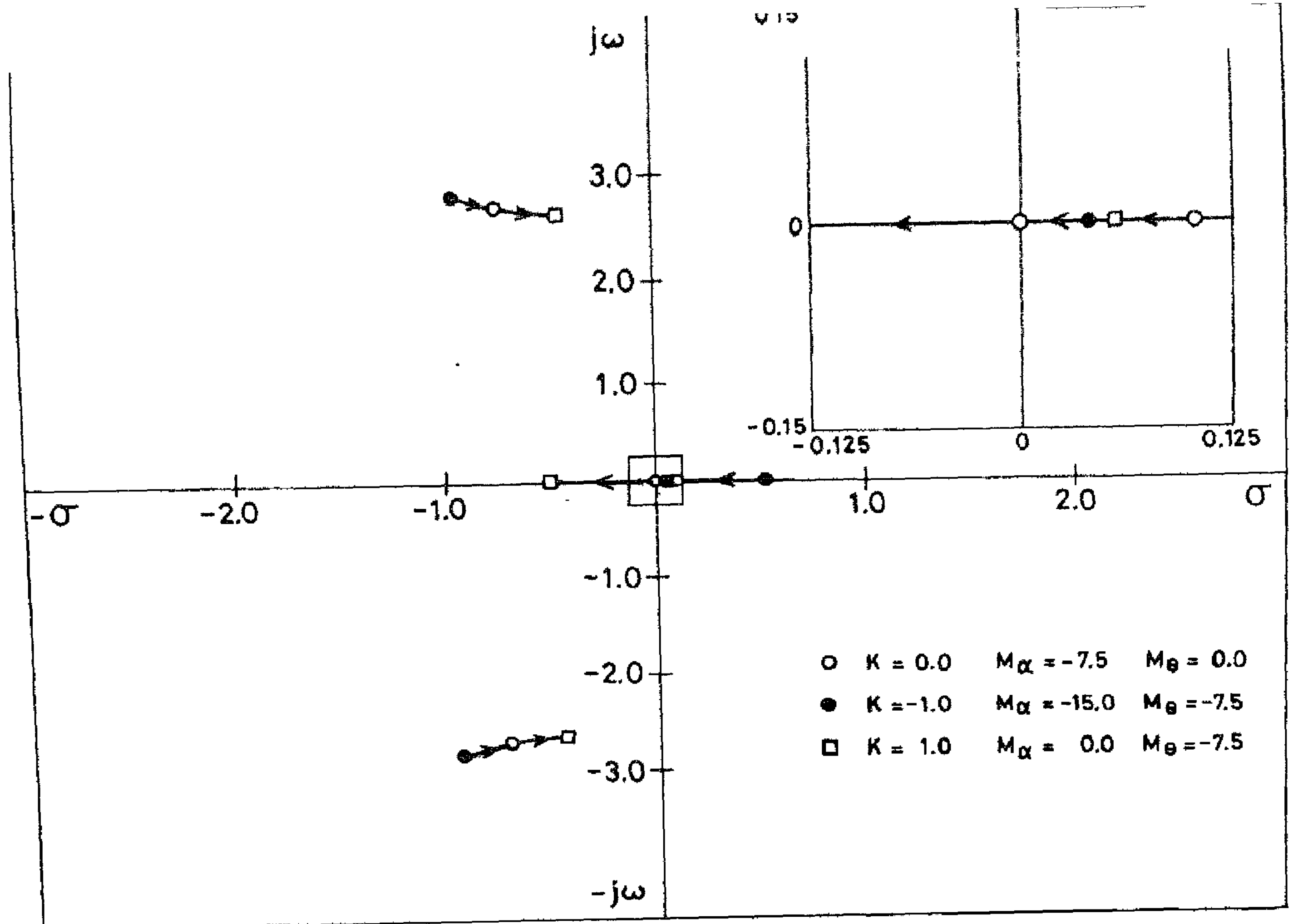
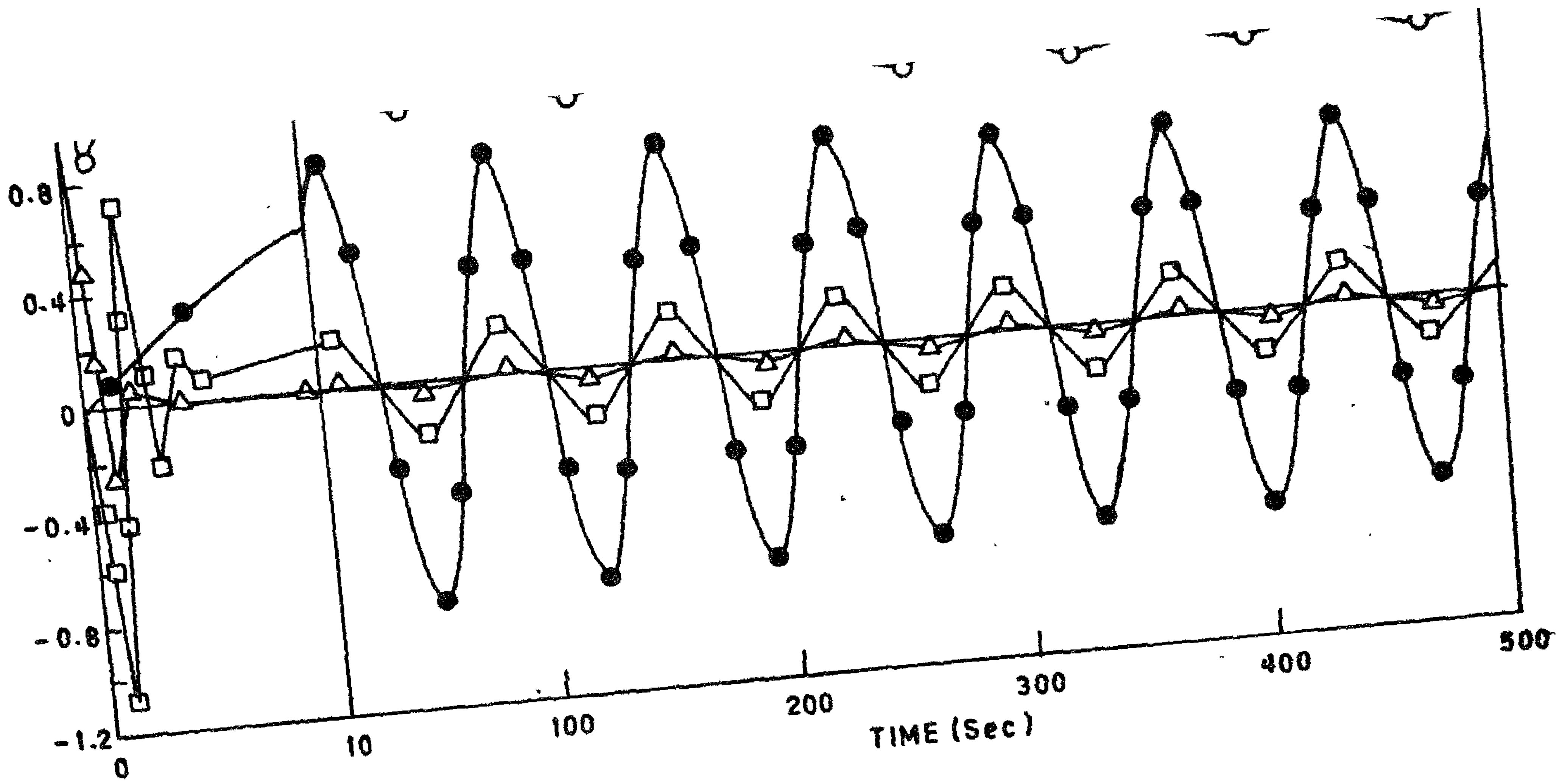
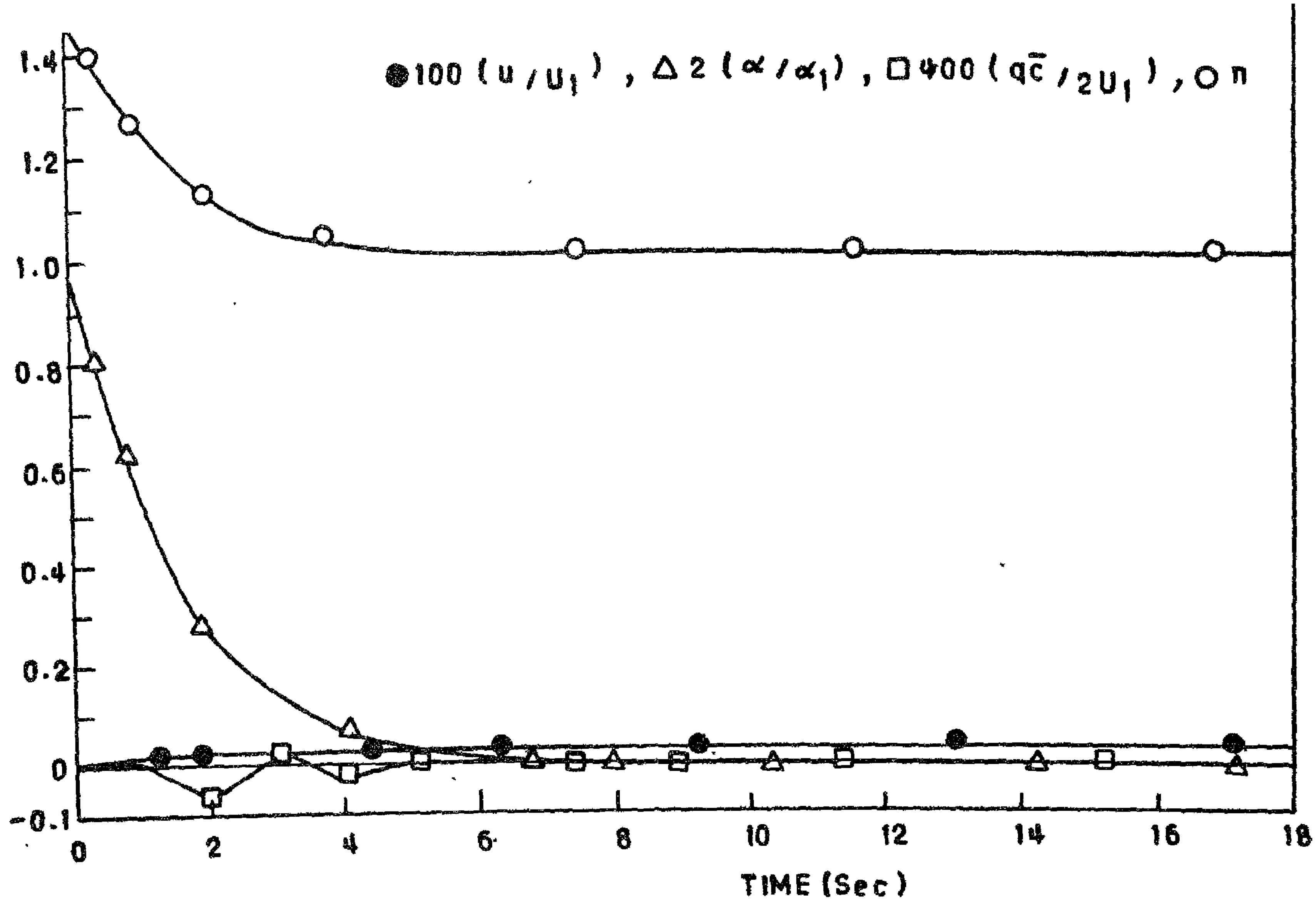


Fig. 3·8 Root locus - variation with K in dive at 60°.



0.8
0.4
0
-0.4
-0.8
-1.2
0
10
100
200
300
400
500
TIME (Sec)


●100 (u/U1), △2 (α/α1), □400 (qc̄/2U1), ○n
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
-0.1
0
2
4
6
8
10
12
14
16
18
TIME (Sec)

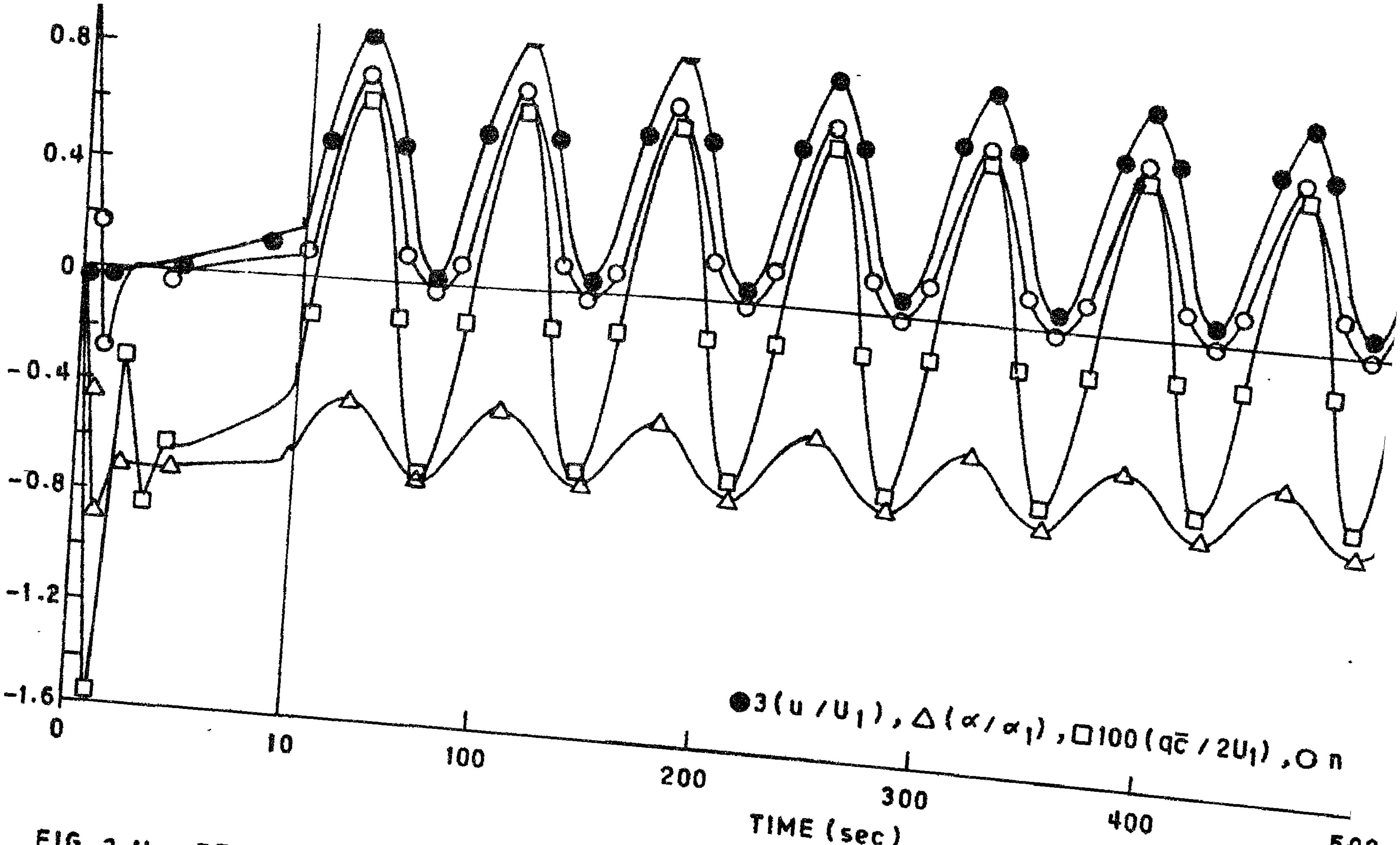


FIG. 3.11 RESPONSE TO UNIT STEP ELEVATOR IN CRUISE FOR CONVENTIONAL AIRCRAFT ($M_\alpha = -7.5$, $M_\theta = 0.0$)

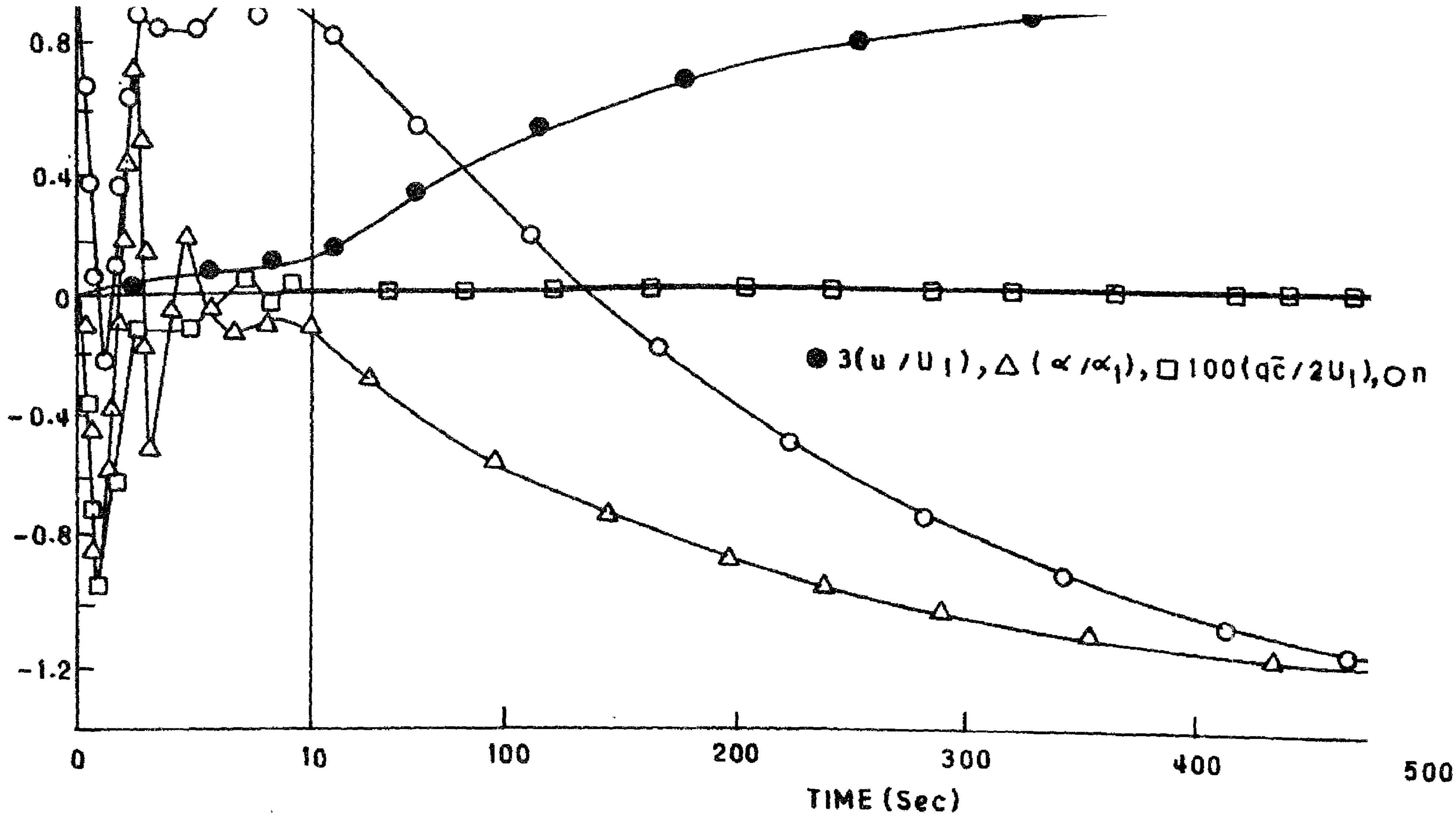


FIG. 3.12 RESPONSE TO UNIT STEP ELEVATOR IN CRUISE FOR SUPERAUGMENTED AIRCRAFT
($M_{\alpha} = 0.0$, $M_{\theta} = -7.5$)

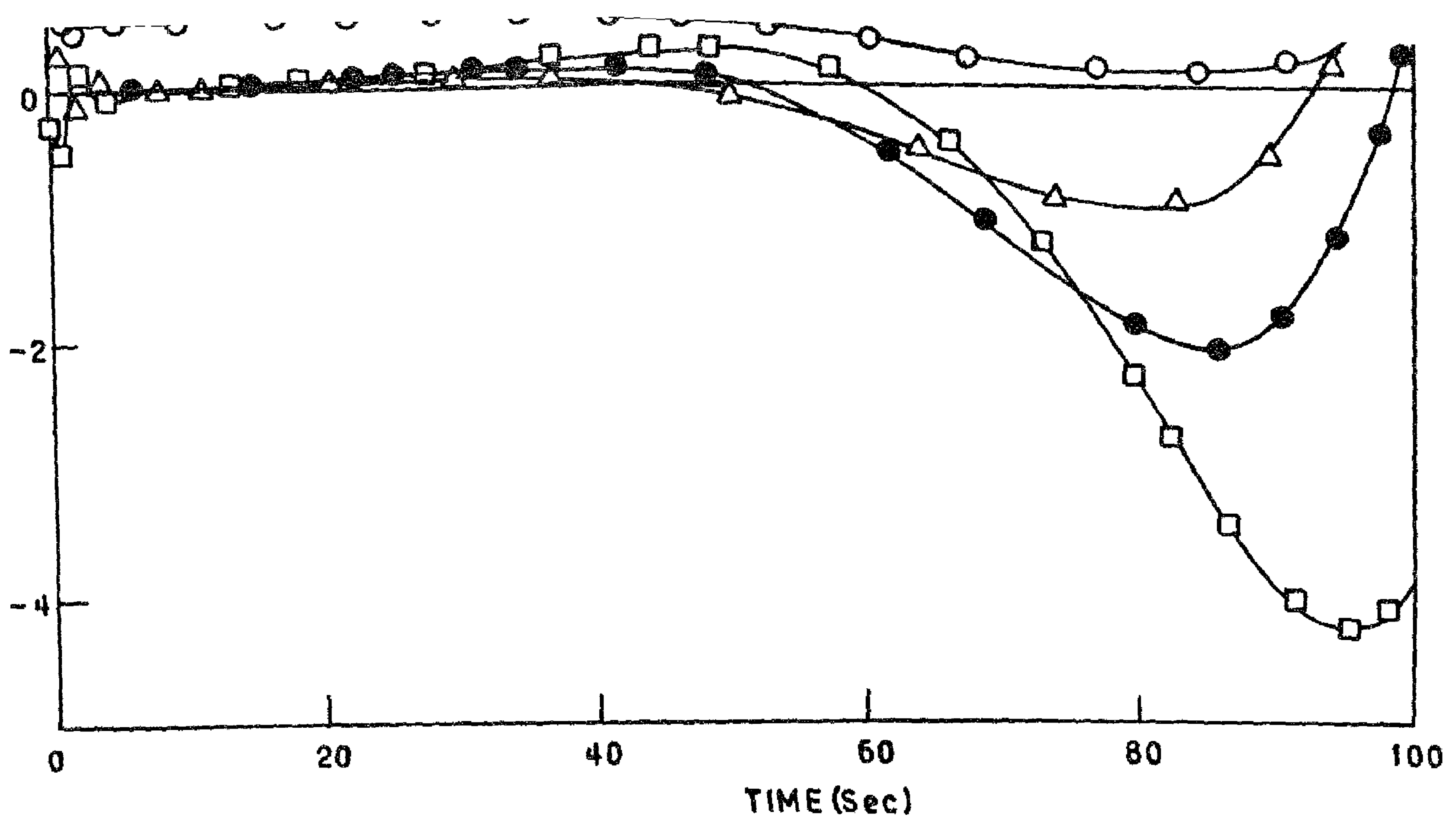


FIG. 3.13 RESPONSE TO UNIT ANGLE OF ATTACK DISTURBANCE IN CLIMB AT 60° FOR

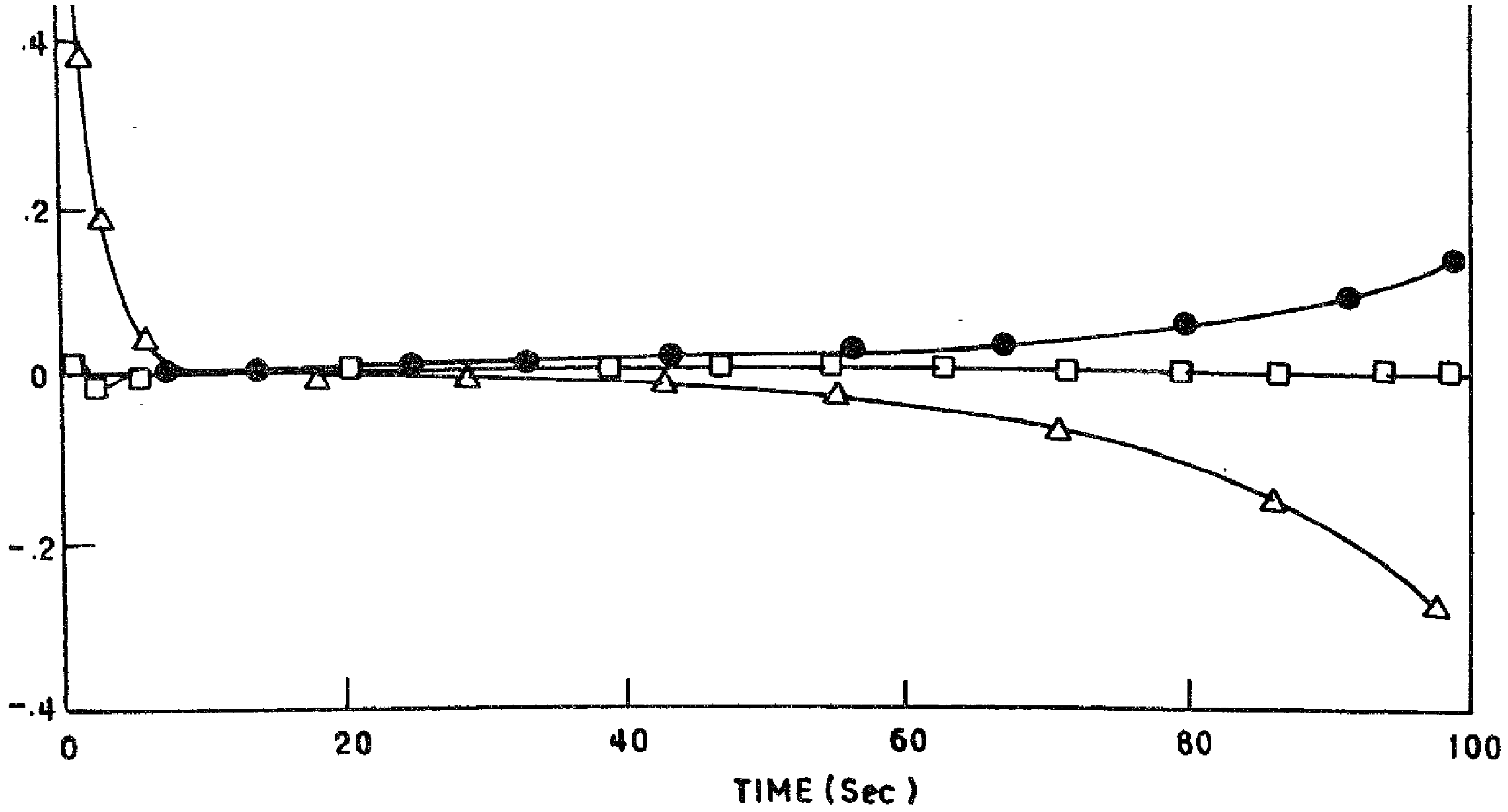
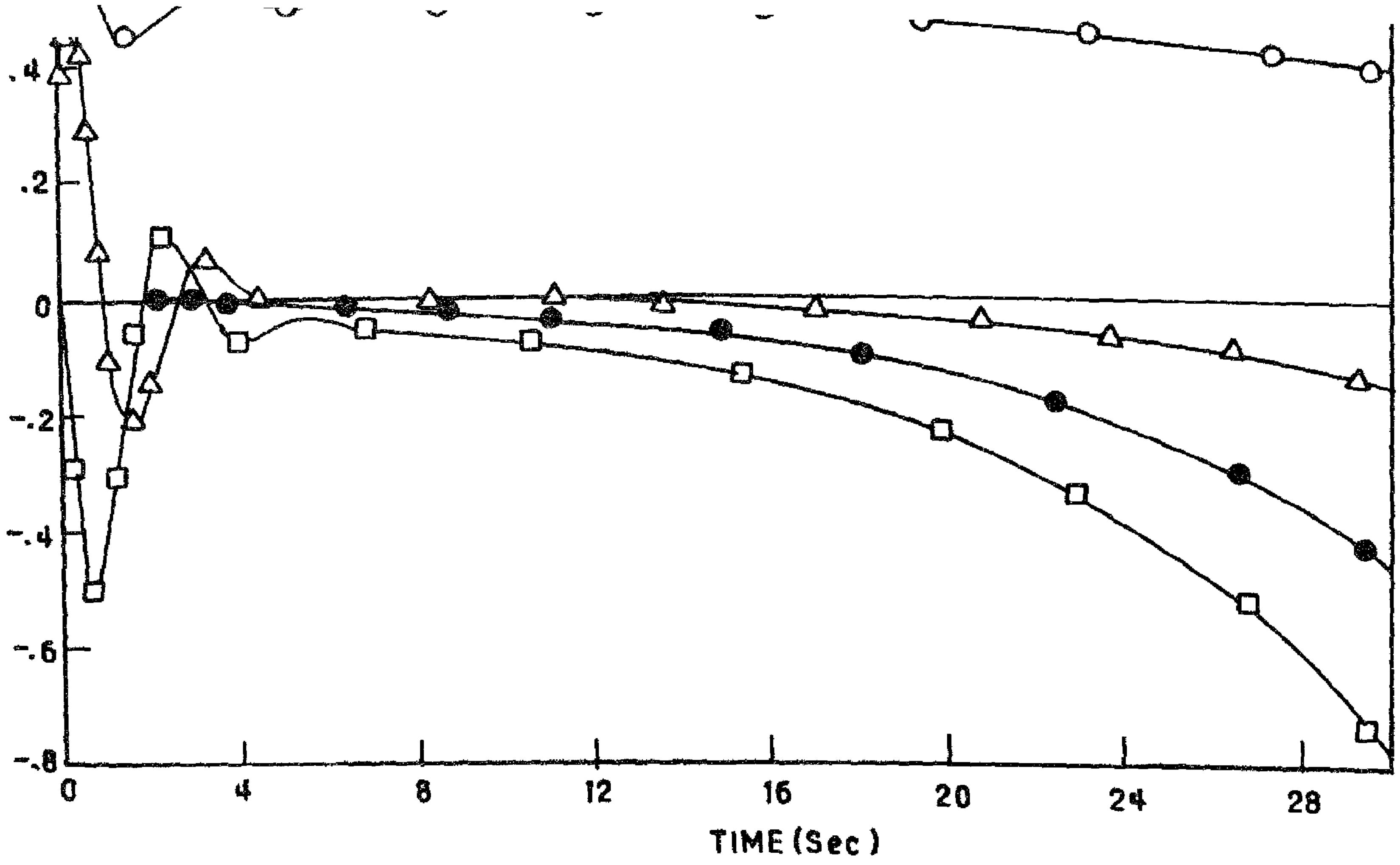


FIG. 3.14 RESPONSE TO UNIT ANGLE OF ATTACK DISTURBANCE IN CLIMB AT 60° FOR SUPERAUGMENTED AIRCRAFT ($M_\alpha = 0.0, M_\theta = -3.724$)


.4
.2
0
-.2
-.4
-.6
-.8
0
4
8
12
16
20
24
28
TIME (Sec)

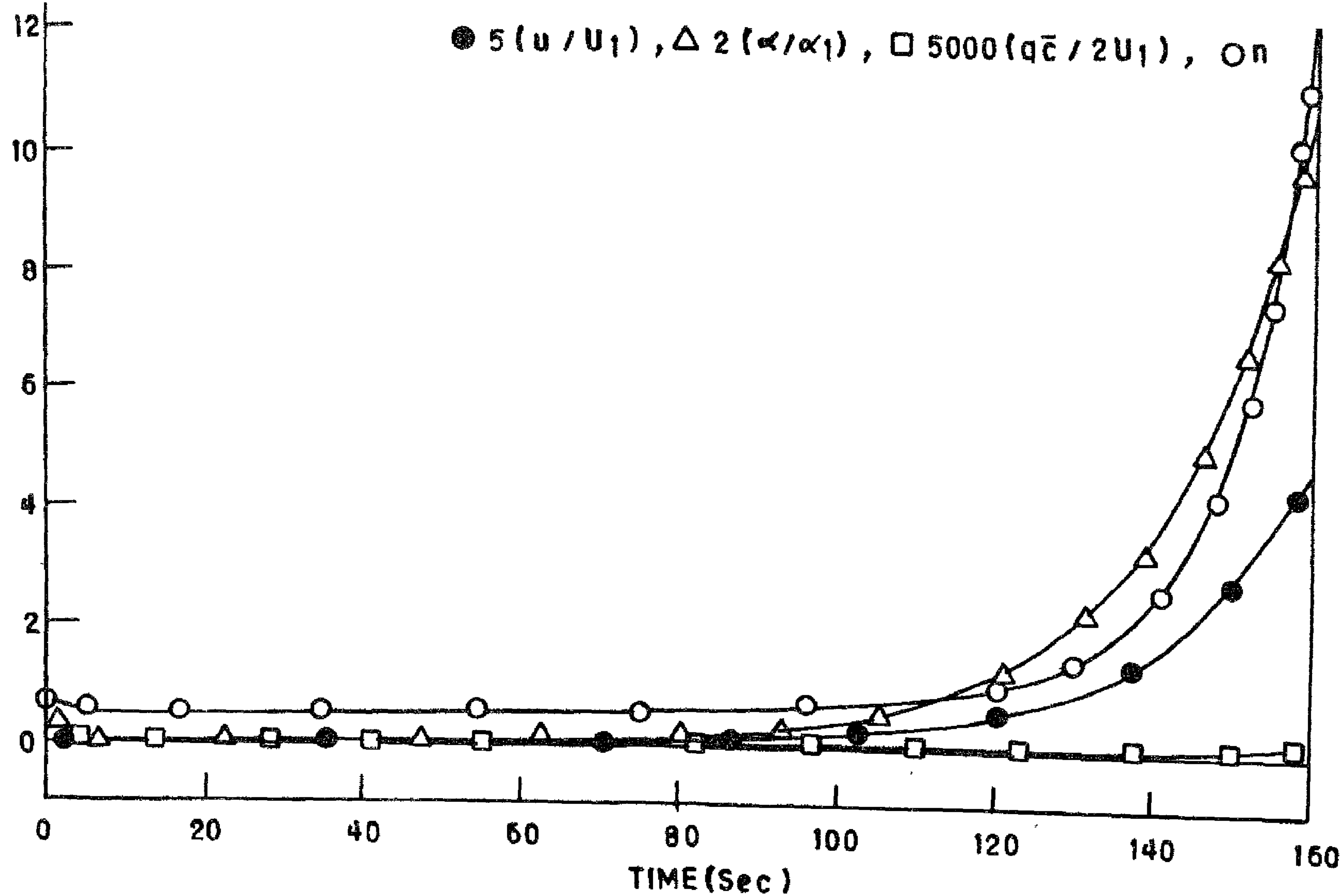


FIG. 3.16 RESPONSE TO UNIT ANGLE OF ATTACK DISTURBANCE IN DIVE AT 60° FOR SUPERAUGMENTED AIRCRAFT ($M_\alpha = 0.0, M_\theta = -3.724$)

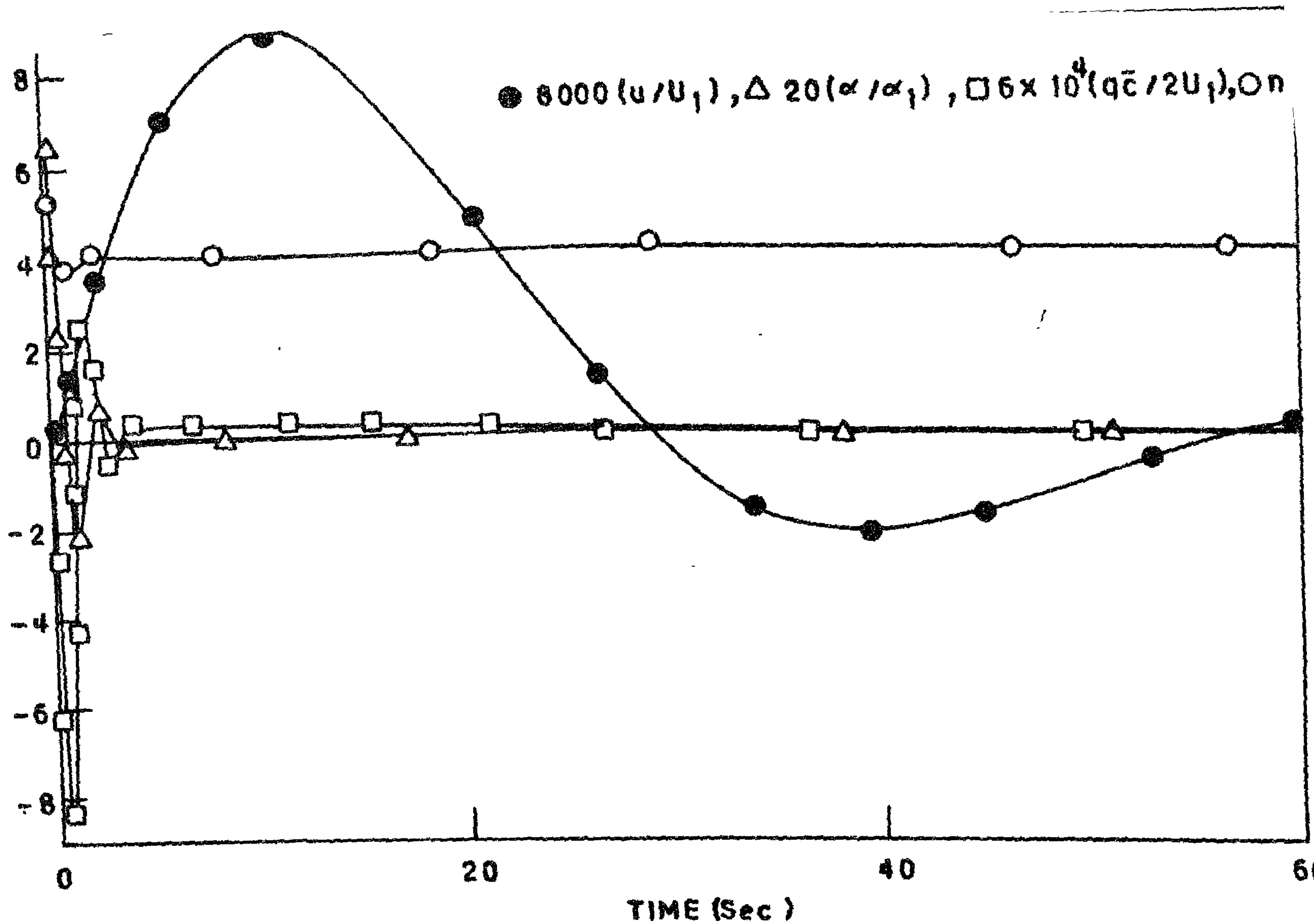
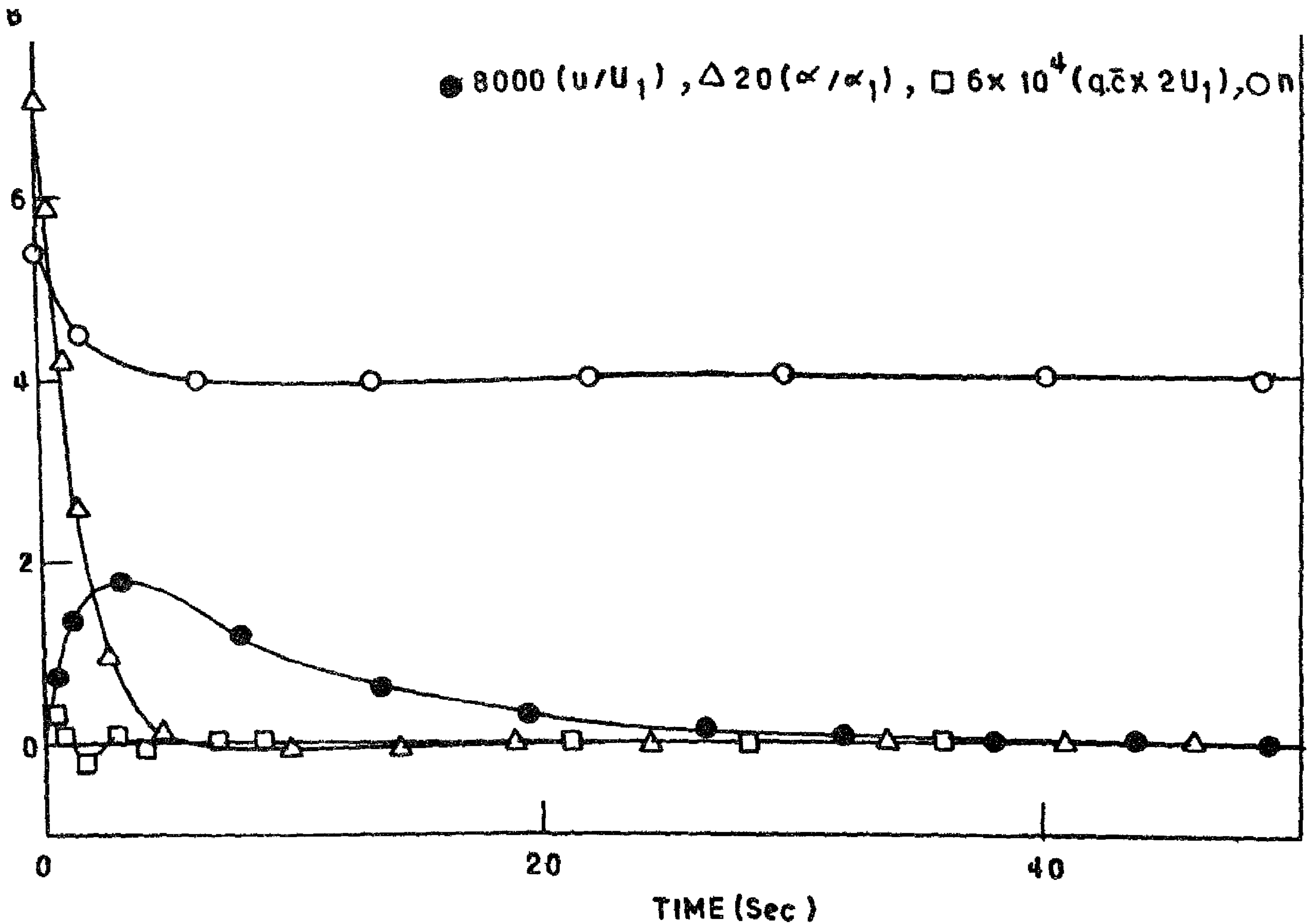
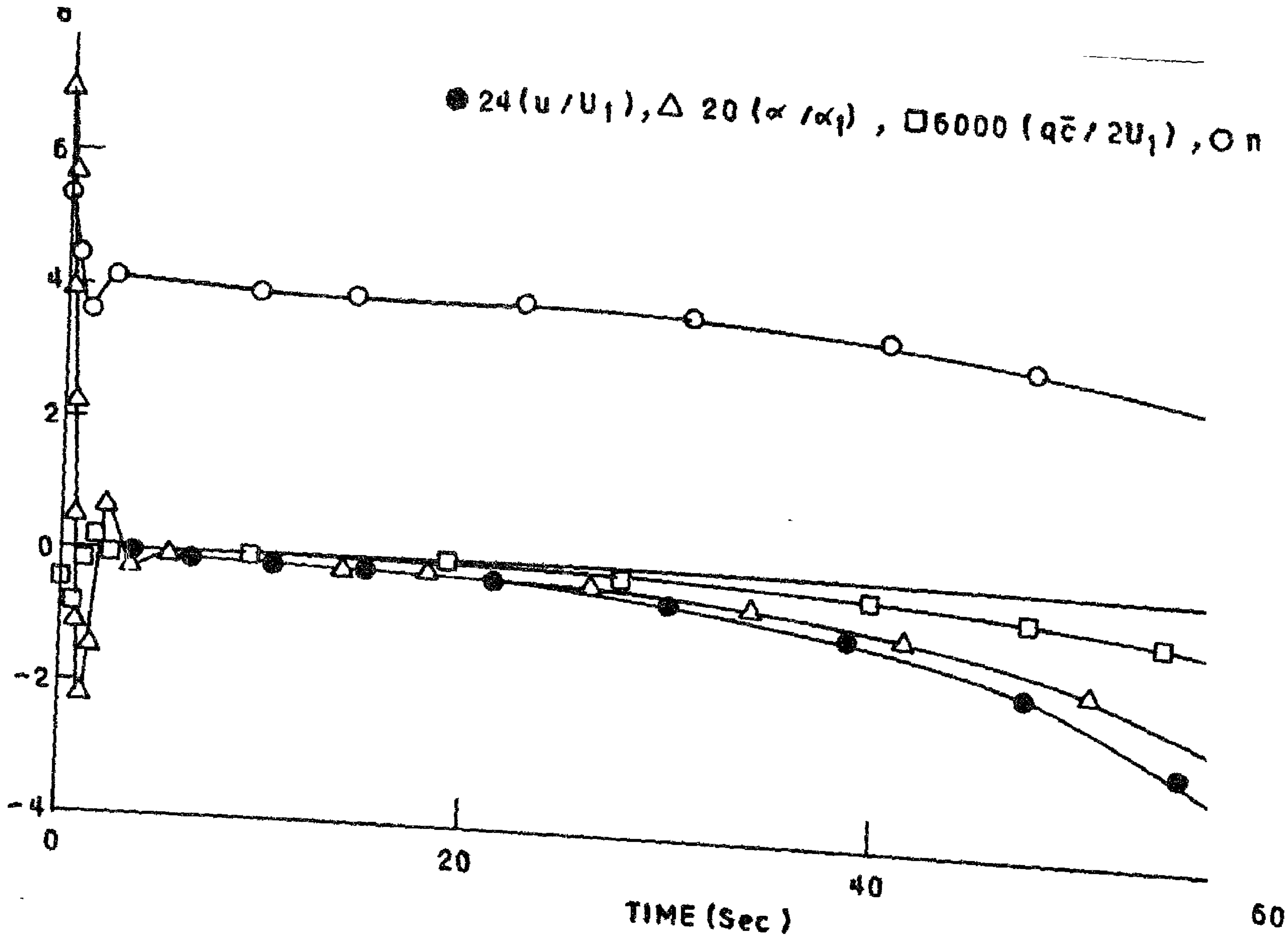


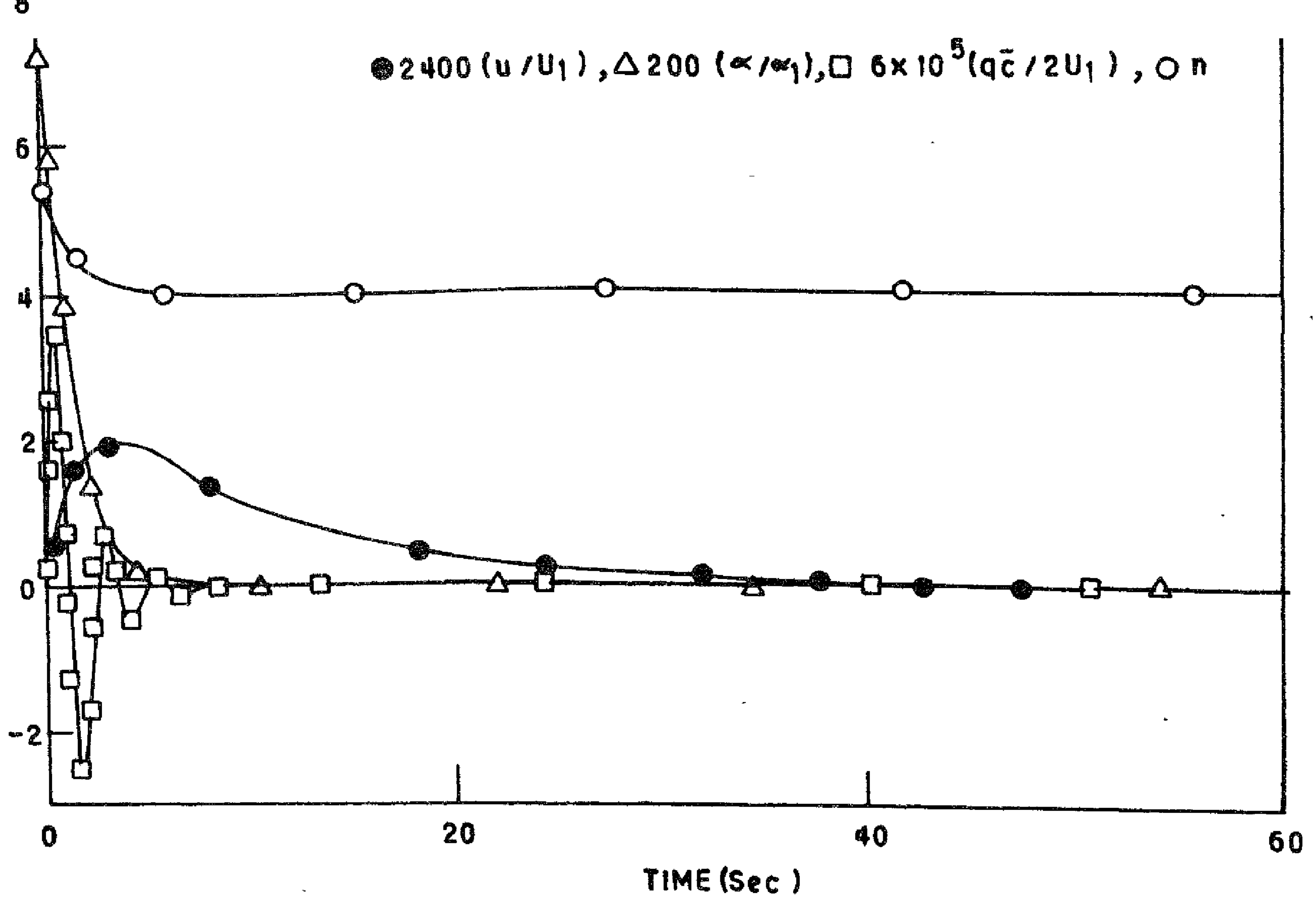
FIG. 3.17 RESPONSE TO UNIT ANGLE OF ATTACK DISTURBANCE IN A VERTICAL LOOP MANEUVER OF LOAD FACTOR 4 AND CONSTANT SPEED AT THE BOTTOM MOST POINT FOR CONVENTIONAL AIRCRAFT ($M_\alpha = -7.5$ $M_\theta = 0.0$)



8000 (u/U1), Δ 20 (α/α1), □ 6x 10^4 (q.c̄ x 2U1), ○ n
0
2
4
6
8
0
20
40
TIME (Sec)



8
6
4
2
0
-2
-4
0
20
40
60
TIME (Sec)
● 24 (u / U1), △ 20 (α / α1), □ 6000 (qc̄ / 2U1), ○ n



●2400 (u / U1), △200 (α / α1), □ 6×10^5 (qc̄ / 2U1), ○ n
8
6
4
2
0
-2
0
20
40
60
TIME (Sec)

APPENDIX A

Expressions for the Coefficients of the Characteristic Equation of Longitudinal Motion

The characteristics equation of the controls fixed longitudinal dynamics of a rigid aircraft is given by

$$f(s) = \sum_{i=0}^{N=4} C_i s^i = As^4 + Bs^3 + Cs^2 + Ds + E = 0.$$

The coefficients of the characteristic polynomial are given by the following expressions:

I. DIMENSIONAL FORM

$$A = U_1 - Z_{\dot{\alpha}}$$

$$B = -M_q(U_1 - Z_{\dot{\alpha}}) - Z_\alpha - (X_u + X_{T_u})(U_1 - Z_{\dot{\alpha}}) - (Z_q + U_1)M_{\dot{\alpha}}$$

$$C = -M_\theta (U_1 - Z_{\dot{\alpha}}) + Z_\alpha M_q + (X_u + X_{T_u})(U_1 - Z_{\dot{\alpha}}) M_q$$

$$+ (X_u + X_{T_u})Z_\alpha - (M_\alpha + M_{T_\alpha})(Z_q + U_1) + (X_u + X_{T_u})(Z_q + U_1)M_{\dot{\alpha}}$$

$$- X_\alpha Z_u$$

$$D = Z_\alpha M_\theta + (X_u + X_{T_u})(U_1 - Z_{\dot{\alpha}}) M_{\dot{\theta}} - (X_u + X_{T_u})Z_\alpha M_q$$

$$+ (X_u + X_{T_u})(Z_q + U_1)(M_\alpha + M_{T_\alpha}) + X_\alpha Z_u M_q$$

$$-X_\alpha (Z_q + U_1)(M_u + M_{T_u}) + g Z_u M_{\dot{\alpha}} + g(M_u + M_{T_u})(U_1 - Z_{\dot{\alpha}})$$

$$E = -(X_u + X_{T_u}) Z_\alpha M_\theta + X_\alpha Z_u M_\theta + g Z_u (M_\alpha + M_{T_\alpha})$$

$$-(M_\alpha + M_{T_\alpha})(X_u + X_{T_u}) - g(M_u + M_{T_u}) Z_{\dot{\alpha}}$$

II. <u>NON-DIMENSIONAL FORM</u>

$$f(s) = \bar{A}s^4 + \bar{B}s^3 + \bar{C}s^2 + \bar{D}s + \bar{E} = 0$$

$$\bar{A} = 2\mu i_B (2\mu - c_{z_{\dot{\alpha}}})$$

$$\bar{B} = -2\mu i_B(c_{z_\alpha} + c_{x_u}) + i_B(c_{x_u} c_{z_{\dot{\alpha}}}) - 2\mu (c_{z_q} c_{m_{\dot{\alpha}}} - c_{m_q} c_{z_{\dot{\alpha}}}) - 4\mu^2 (c_{m_{\dot{\alpha}}} + c_{m_q})$$

$$\bar{C} = i_B(c_{x_u} c_{z_\alpha} - c_{x_\alpha} c_{z_u}) + 2\mu(c_{z_\alpha} c_{m_q} - c_{m_\alpha} c_{z_q} + c_{x_u} c_{m_q} + c_{x_u} c_{m_{\dot{\alpha}}}) - 4\mu^2 c_{m_\alpha} - 4\mu^2 c_{m_\theta}$$

$$+ 2\mu c_{z_{\dot{\alpha}}} c_{m_\theta} - c_{x_u} (c_{m_q} c_{z_{\dot{\alpha}}} - c_{z_q} c_{m_{\dot{\alpha}}}) + 2c_{L_o} c_{x_\alpha} i_B$$

$$\bar{D} = -2 c_{L_o}^2 c_{m_{\dot{\alpha}}} + 2\mu(c_{x_u} c_{m_\alpha} - c_{x_\alpha} c_{m_u} + c_{L_o} c_{m_u})$$

$$+(-c_{z_\alpha} + c_{x_u}) c_{m_\theta} + c_{x_u} (c_{m_\alpha} c_{z_q} - c_{m_q} c_{z_\alpha} - c_{z_{\dot{\alpha}}} c_{m_\theta})$$

$$-c_{x_\alpha} (c_{m_u} c_{z_q} - c_{m_q} c_{z_u}) - c_{L_o} (c_{m_u} c_{z_{\dot{\alpha}}} - c_{z_u} c_{m_{\dot{\alpha}}})$$

$$-2c_{L_o} c_{m_q} c_{x_\alpha}$$

$$\bar{\bar{E}} = -c_{L_o}\left(c_{m_o}\left(2c_{L_o} - c_{z_u}\right) + c_{m_u} c_{z_\alpha}\right)$$

$$+ c_{m_\theta}\left(-c_{x_u} c_{z_\alpha} + 2c_{x_\alpha} c_{L_o} + c_{x_\alpha} c_{z_u}\right)$$

```
THIS PROGRAM SOLVES NUMERICALLY THE LONGITUDINAL PERTURBATION
EQUATION OF MOTION OF AIRCRAFT,GIVEN THE NON-DIMENSIONAL AERODYN
A IC DERIVATIVES,GEOMETRIC CONFIGURATION AND FLIGHT CONDITIONS
AND INITIAL CONDITIONS.
IT CALLS THE ROUTINE D02BBF OF STANDARD NAG LIBRARY WHICH
USES RUNGE-KUTTA MERSON METHOD TO SOLVE THE
ORDINARY DIFFERENTIAL EQUATION
------------------------------------------------------------------
N IS THE NUMBER OF DEPENDENT VARAIBLE.W IS THE WORKING SPACE OF
DIMENSION (N,8),257,X IS THE INITIAL VALUE OF INDEPENDENT
VARIABLE,XEND IS THE FINAL VALUE OF INDEPENDENT VARIABLE.
IRELAB IS EQUAL TO 0,1 ACCORDING TO THE TYPE OF ERROR CONTROL
Y(N) IS THE OUTPUT AT INTERMEDIATERY POINTS,IFAIL IS THE ERROR
CHECK AND UNLESS THE ROUTINE DETECTS AN ERROR,IFAIL CONTAINS
0 ON EXIT.SUBROUTINE FCN IS GIVEN THE SET OF DIFFERENTIAL
EQUATIONS TO BE SOLVED.SUBROUTINE OUTPUT IS GIVEN TO HAVE
ACCESS TO INTERMEDIATE VALUES OF COMPUTED SOLUTION.
==================================================================
X-INTERMEDIATE POINT OF INDEPENDENT VARIABLE
OUTPUT:U=Y(1),ALPHA=Y(2),THETA=Y(3),Q=Y(4),ALDF2=LOADFACTOR
GAM1,GAM ARE THE FLIGHT ANGLE
==================================================================
DIMENSIONS TO BE DEFINED IN ACCORDANCE WITH NAG ROUTINE D02BBF
N=4,NO:OF VARIABLE,M=4*N
==================================================================
DIMENSION W(4,9),Y(4),GAM(500),GAM1(500),ALDF2(500),AY(4)
EXTERNAL FCN,OUT
COMMON XEND,H,T
OPEN(UNIT=26,DEVICE='DSK',FILE='DO26.OUT')
OPEN INPUT OUTPUT FILES.INPUT FILE CONTAINS FLIGHT CONDITIONS
NON DIMENSIONAL-AERODYNAMIC DERIVATIVES,GEOMETRIC CONFIG
OPEN(UNIT=38,DEVICE='DSK',FILE='DO38.INP')
READ(38,*) CD1,CDU,CTXU,CTX1,CDALP,CL1,CDDEL,CLU,CL1,CLALP,CD1
1 CLALPD,CL [illegible],CMU,CM1,CMTU,CMT1,CMALP,CMTALP,CMALPD,CMQ,CMD
2 [illegible],U1,S,DENSIT,C,IYY,G
Q1=.5*U1**2*DENSIT
M=W/G
CALCULATION OF DIMENSIONAL AERODYNAMIC DERIVATIVES
XU=-Q1*S*(C  +2*C  ))/(M*U1)
X1  =Q1*S(CTXU+2*CTX1)/(M*U1)
```

```
[illegible]
DIMENSION [illegible] Y(4)
CONSTANTS [illegible]
F*C**2[illegible]/[illegible]*IYY[illegible]
[illegible]=0.1*[illegible]/IYY
N=4
IRELAB=0
ERROR CONTROL THRU TOL
DO 20 I=5,6
TOL=10.**(-I)
WRITE(36,90)TOL
WRITE(36,99)
RANGE OF TIME FOR WHICH OUTPUT IS OBTAINED
X=0.
XEND=500.
INITIAL CONDITIONS OF DEPENDENT VARIABLE
Y(1)=0.
ANGLE OF ATTACK PERTURBATION
Y(2)=0.0174532
Y(3)=0.0
Y(4)=0.0
H=(XEND-X)/500
I=100.
IFAIL=1
CALL D02BBF(X,XEND,N,Y,TOL,IRELAB,FCN,OUT,W,IFAIL)
WRITE(36,97)IFAIL
IF(IFAIL.EQ.0)WRITE(36,95)


CONTINUE
FORMAT(2X,'TOL=',E9.1)


FORMAT(2X,'X AND SOLN')
FORMAT(1X,'IFAIL = ',I3)
FORMAT(1X,'RANGE TOO SMALL')
STOP
END

SUBROUTINE FCN(T,Y,F)
DIMENSION F(1),Y(4)
```

```
F(2)=(-[illegible]*(5)*(Z)*Y(1)+ZALPHA*Y(2)+(Z0+[illegible])*Y(3)+ZDE
[illegible]
F(3)=Y(4)
F(4)=PHI*Y([illegible])+[illegible]*Y(1)+MALPHA*Y(2)+MTALPH*Y(2)+MALPHD*F(2)+MG*Y(3
[illegible]
RETURN
END
OUTPUT ROUTINE
SUBROUTINE OUT(X,Y)
DIMENSION Y(4),GAM(500),GAM1(500),ALPF2(500),AY(4)
COMMON XEND,H,I
AY(2)=3*Y(2)*57.295779/(2.07)
AY(4)=20.*(Y(4)*57.295779*7.04/(2.0*H1))
AY(1)=(Y(1)/H1)*[illegible]
NX=X+1
ALPF2(NX)=((H1+Y(1))**2.0*(ALPHA1+Y(2)))/(H1**2.
3 0*ALPHA1))
GAM(NX)=Y(3)-Y(2)
GAM1(NX)=57.295779*GAM(NX)/2.07
WRITE(6,999)X,AY(1),AY(2),AY(4),ALPF2(NX)
FORMAT(1X,F7.[illegible],2(1X,F15.4))
X=XEND-FLOAT(I)*H
I=I-1
FORMAT(1X,F7.2,5F13.5)
RETURN
END
```

```
[illegible] ITS OF POLYNOMIA WHOSE ORDER IS N
[illegible] EFFICIENTS [illegible] EAL USING STANDARD NAG ROUTINE C02AEF
-------------------------------------------------------------------
[illegible] A(50),REZ(50),IMZ(50),TOL,N)

[illegible]
[illegible] FILE=ROOT.INP,OUTPUT FILE=ROOT.OUT
[illegible] COEFFICIENTS OF POLYNOMIAL
OPEN(UNIT=6,DEVICE='DSK',FILE='ROOT.OUT')
OPEN(UNIT=20,DEVICE='DSK',FILE='ROOT.INP')
N IS ORDER OF POLYNOMIAL
N=5
PARAMETER VARIED
[illegible]=0.0
INPUT IN TERMS OF NON DIMENSIONAL DERIVATIVES,FLIGHT CONDITION
AND GEOMETRIC CONFIG
READ(20,*) CDU,CD1,CTXU,CTX1,CDALP,CL1,CDDEL,CLU,CL1,CLALP,CD1
1 CLALPD,CLQ,CLDEL,CMU,CM1,CMTU,CMT1,CMALP,CMTALP,CMALPD,CMQ,CMD
2 EL,W,U1,S,DENSIT,C,IYY,G
Q1=.5*U1**2*DENSIT
M=W/G
[illegible] OF DIMENSIONAL DERIVATIVE
X =-(Q1*S*(CDU+2*CD1))/(M*U1)
X1U=Q1*S*(CTXU+2*CTX1)/(M*U1)
[illegible]=Q1*S*(CDALP-CL1)/M
XDEL=-Q1*S*CDDEL/M
Z =-Q1*S*(CLU+2*CL1)/(M*U1)
ZALP=Q1*S*(CLALP+CD1)/M
ZALPD=Q1*S*C*CLALPD/(2*M*U1)
ZQ=-Q1*S*C*CLQ/(2*M*U1)
ZDEL=-Q1*S*CLDEL/M
M =Q1*S*C*(CMU+2*CM1)/(IYY*U1)
MTU=Q1*S*C*(CMTU+2*CMT1)/(IYY*U1)
MALP=Q1*S*C*CMALP/IYY
MTALP=Q1*S*C*CMTALP/IYY
MALPD=Q1*S*C*C*CMALPD/(2*IYY*U1)
MQ=Q1*S*C*C*CMQ/(2*IYY*U1)
MDEL=Q1*S*C*CMDEL/IYY
EVAL[illegible] CHARACTERISTIC COEFFICIENTS
A(1)=[illegible]
```

```
1     + T )*   -/   P ))+(    + T )*[-XALP*([1+ZO)]+ZJ*XALP*MQ+(XU+XTU)
2 )*[(  A  )  + Z*(  A)*()1+ZO)-MQZALP]
A(5)  J*CO  (  1)*[(  A PHA+  TALP)*ZU-ZALP*(MJ+MTU)]+Q*SIN(01)*[(MT+
3   1  )*XA   P-(X .+XT )*JALPHA+MIALP)
IFAIL=0
FINDS ROOTS WHEN TOL
TOL=0.01E-4
CALL C02AEF(A,N,REZ,IMZ,TOL,IFAIL)
WRITE(6,20)MQ,(REZ(M),IMZ(M),M=1,4)
TYPE *,A(1),A(2),A(3),MQ
FORMAT(1X,F7.3,8(F15.4))
MQ=MQ+0.05
IF(MQ.GT.1.0)GO TO 40
N=5
GOTO 50
WRITE(6,*)IFAIL,N
MQ=1.0
STOP
END
```